Music 421A
Spring 2019-2020
Homework #3
Windows

Theory Problems

1. (5 pts) **$\cos^p$ Window**
   Show that the $\cos^p(t)$ window has p leading zeros in the series expansion about its right endpoint. What is its roll-off rate in dB per octave?

2. (10 pts) **System Identification Using the Poisson Window**
   Suppose we are attempting to measure and model a *loudspeaker* impulse response $h(t)$. We want to model the speaker impulse response using a one-pole filter:
   $$y(n) = b_0\, x(n) - a_1\, y(n-1)$$
   where, b_0, and a_1 are our design parameters we will be fitting.

   Suppose we record the speaker impulse response using a sampling rate $f_s = 50,000$ Hz, and the impulse response is measurably nonzero for more than 2 minutes due to the extreme low-frequency response supported. We have limited memory in which to load the file for our model fitting, such that we can only store 2 seconds of data.

   (a) (5 pts) We want to estimate the model using the first 2 seconds of data using the right half of a Poisson window having a t_{60} of the same length (2 seconds). What is the α parameter of the Poisson window that will achieve this specification? Recall that the Poisson window is defined as
   $$w_P(n) = w_R(n) e^{-\alpha \frac{|n|}{\frac{M-1}{2}}}.$$

   (b) (5 pts) Suppose we estimate the pole of our model to be $a_1 = p \in [0, 1)$. What is the corrected pole location obtained by removing its artificial contraction due to the Poisson window?

3. (10 pts) **Chebyshev Window**

   (a) (5 pts) Let $W(\omega_a) = T_M(\alpha\omega_a)$, $\omega_a \in (-\infty, \infty)$, be a continuous spectrum, where T_M denotes the Mth Chebyshev polynomial, and let α be any constant scale factor. Show that the corresponding time-domain signal $w(t)$ has finite support, and find the range of t specifying that support.

   (b) (5 pts) Let $W(\omega_d) = T_M[\alpha\cos(\omega_d/2)]$, $\omega_d \in (-\pi, \pi)$, be the continuous spectrum (DTFT) of a discrete-time signal $w(n)$, where T_M again denotes the Mth Chebyshev polynomial, and α is any constant scale factor. Show that the corresponding time-domain signal $w(n)$ has finite support, and find the integers n specifying that support.

Lab Problems

1. (15 pts) **Kaiser Window**
   For definiteness below, let the Kaiser-window length be $M = 151$.
   (a) (5 pts) Find to three significant digits the value of the Kaiser-window β parameter that achieves at least 80 dB side-lobe attenuation.
   (b) (5 pts) For this value of β, compute the estimated first null frequency in cycles/sample.
   (c) (5 pts) Plot the magnitude spectrum of this window, marking the -80 dB side-lobe level with a horizontal line, and the calculated null frequency with a vertical line.

2. Suppose you have a periodic signal $x(n)$ having many harmonics and an FIR lowpass filter having a rectangular impulse response $h(n) = w_R(n)$:
   (a) (3 pts) What length M should be chosen for the FIR filter to eliminate the fundamental frequency f_0 and all harmonics $k\,f_0$ up to half the sampling rate $f_s/2$ in the mixture?
   (b) (3 pts) What if the filter impulse response is now a triangular function as defined in HW1?
   (c) (2 pts) In MATLAB, generate a sum of cosines having frequencies 400,800,1200 Hz and 300,600,900 Hz, i.e., the sum of two harmonic sources of fundamental frequencies 400 and 300 Hz. Use the sampling frequency $f_s = 8$kHz and the signal duration of 1 second.
   (d) (2 pts) Find the value of M of a rectangular impulse response which will get rid of the harmonics of the source with the fundamental frequency of 400 Hz. Construct the rectangular impulse response of the filter.
   (e) (3 pts) Show in the time-domain (by convolution) or the frequency-domain (by FFT) that all harmonics of the 400 Hz source are more or less gone. Plot the spectrum of the original mixed signal as well as the signal after filtering. Also, verify the result by listening. (You can use `soundsc(x,fs)` to listen to a signal in Matlab.) Is it what you expected from the plots?
   (f) (Optional) Replace the signal consisting of multiples of 300 Hz by a soundfile of your choice. Listen to three cases: (1) the original soundfile, (2) the sum of the soundfile plus the tone having multiples of 400 Hz, and (3) result of filtering using the rectangular window that suppresses the added tone. Can you hear any residual added tone in the result? Is the "restored" soundfile distorted in any way? Share your results with your classmates and listen to their results.