Music 421A
Spring 2019-2020
Homework #2
Windows and Fourier Theorem Review
Due in one week

Theory Problems

1. (4 pts) Consider the 8 point DFT of some arbitrary input, where the sampling rate is 3 Hz. Write the frequency in Hz represented by every point of the DFT.

2. (5 pts) For the sequence $x(n) =$ `[1:8,8:-1:1]` (matlab notation), sketch or plot $\text{STRETCH}_2(x)$ and its magnitude spectrum.

3. (5 pts) **Piano Key Resolution:** The low A key on a modern piano is tuned to A0 which is 27.5 Hz (four octaves below A440). Find the length of rectangular window (in seconds) necessary to resolve the fundamental frequency of this note relative to the adjacent note at Bb0. Repeat for middle C, which is at 261.63 Hz and C# one key above middle C. [Hint: You may assume that the piano is tuned according to standard equal-temperament tuning with 12 uniform semitones per octave.] Comment on how fast a piano performance can be in each frequency range and still be able to transcribe it reliably.

4. (5 pts) You are given a real windowed sinusoid of the following form:

$$x(t) = w(t)\cos(\omega t + \phi).$$

   The window function $w(t)$ and sinusoidal phase ϕ are unknown. Show how you can remove the sinusoidal component $\cos(\omega t + \phi)$ in two steps to recover the unknown window. For the first step, use properties of sinusoids to convert $x(t)$ into an analytic signal $z(t)$ (having only positive frequencies). Assume the total bandwidth of w (including both positive and negative frequencies) is less than 2ω. Once in analytic form, use common signal processing operators to "separate" or demodulate the window function from the sinusoid. Run a small simulation in matlab showing the procedure.

5. (5 pts) Derive the expression and sketch the window $w(n)$ corresponding to the window transform

$$W(\omega) = \frac{1}{2}\text{asinc}_M(\omega + \omega_1) + \text{asinc}_M(\omega) + \frac{1}{2}\text{asinc}_M(\omega - \omega_1)$$

   where $\omega_1 = 2\pi/M$.

6. (5 pts) Sketch the window $w(n)$ corresponding to the window transform

$$W(\omega) \triangleq -\frac{1}{2}\text{asinc}_M(\omega + \omega_1) + \text{asinc}_M(\omega) - \frac{1}{2}\text{asinc}_M(\omega - \omega_1)$$

   where $\omega_1 = 2\pi/M$ and M is assumed to be odd.

7. (5 pts) Sketch the window $w(n)$ corresponding to the window transform

$$W(\omega) \triangleq e^{j\omega/2}\text{asinc}_M(\omega)$$

   where M is an even integer and $\omega \in [-\pi, \pi)$.

   [Hint: The resulting window is called a "DFT-even" window.]

Lab Assignment

1. In this problem you will write two matlab functions, and at least the second will be useful later in the course. Turn in a copy of your code for each part.

   (a) (3 pts) Write a matlab function that windows and zero-pads an input signal. Your function should take 3 arguments: input signal x to be windowed, window w to be used, and final length N of the zero padded sequence. The lengths of x and w should both be an odd integer M. The code below will get you started:

   ```
   function [output_sig] = zeropadwin(input_sig, window, padded_length)
   % function windows a signal and adds zeros to the end
   % check that input_sig is the same size vector as the window:

   % multiply window by input_sig:

   % add zeros to end so that length(output_sig) is padded_length:
   output_sig =
   ```

   (b) (5 pts) Write a matlab function that **zero-phase** windows and zero-pads an input signal. This is the same as the previous problem, except you must be careful about the order of the output. The output should look similar to Figure 2.6(b) of SASP.[1]

2. In this problem, we consider the question of what happens to the spectrum of a window when we change its length and/or sampling rate. When plotting magnitude spectra in this problem, always normalize for a peak value of 0 dB, and always use `fftshift()` so that frequency 0 is in the center of the plot.

   (a) (2 pts) Plot, using `stem()`, the time domain representation of a 51-sample rectangular window, zero-phase zero-padded to length 256 or more. Double label the time axis in both samples and seconds, assuming a sampling rate of 8192 Hz. How long is your window in seconds?

   (b) (2 pts) Plot the log-magnitude spectrum of the window, and label the frequency axis in Hz. Manually (or using matlab) label the frequency of the first positive-frequency zero crossing in Hz.

   (c) (2 pts) Change the window length to 91 samples and repeat the previous item. How long is this window in seconds?

   (d) (2 pts) Using the window length 91 samples, change the sampling rate to 8000 Hz. How long in seconds is the window now? Plot the log-magnitude spectrum of the window, and label the frequency axis in Hz. Manually label the frequency of the first positive-frequency zero crossing in Hz.

3. (8 pts) Repeat Problem 2 for a Hann window. Compare and contrast the main-lobe width and side-lobe level for analogous plots.

[1] `https://ccrma.stanford.edu/~jos/ReviewFourier/Zero_Centered_Zero_Padding.html`.