

## MUS420 Lecture Piano-Hammer Modeling

Julius O. Smith III (jos@ccrma.stanford.edu)  
Center for Computer Research in Music and Acoustics (CCRMA)  
Department of Music, Stanford University  
Stanford, California 94305

February 5, 2019

### Outline

- Acoustics of the Piano
- The Piano Hammer
- Stulov's Model
- Hammer-String Dynamics

1

## Hammer Modeling

- Introduction:

<https://www.acs.psu.edu/drussell/piano/nonlinearhammer.html>

- Stulov<sup>3</sup> model:

$$f_h(t) = f_0 \left[ u^p(t) - \frac{\epsilon}{\tau_0} \int_0^t u^p(\xi) \exp\left(-\frac{\xi-t}{\tau_0}\right) d\xi \right]$$

where

$f_h(t)$  = force exerted on string by the piano hammer

$u(t)$  = hammer compression

$f_0$  = instantaneous hammer stiffness

$p$  = nonlinearity exponent

$\epsilon$  = hysteresis parameter

$\tau_0$  = hysteresis time constant

- Stulov's piano-hammer model essentially models the properties of the felt (wool)

<sup>3</sup><http://www.cs.ioc.ee/~stulov/>

3

## The Piano

### Acoustics of the Piano

- Five Lectures on the Acoustics of the Piano<sup>1</sup>
- The Piano Hammer as a Nonlinear Spring<sup>2</sup>

<sup>1</sup>[http://www.speech.kth.se/music/5\\_lectures/](http://www.speech.kth.se/music/5_lectures/)

<sup>2</sup><https://www.acs.psu.edu/drussell/piano/nonlinearhammer.html>

2

### Empirically Calibrated Stulov Model

Under normal playing conditions the following approximate Stulov model suffices:

$$f_h(t) = Q_0 \left[ u^p + \alpha \frac{d(u^p)}{dt} \right]$$

where

$$\alpha = 248 + 1.83n - 5.5 \cdot 10^{-2}n^2$$

$$Q_0 = 183 e^{0.045n}$$

$$p = 3.7 + 0.015n$$

$$n = \text{Piano key number } n \in [1, 88]$$

$$u = \text{hammer-felt compression}$$

$$f_h = \text{force of hammer felt against string}$$

with

- $\alpha$  in physical units of microseconds ( $\mu s$ ) and
- $Q_0 = f_0(1 - \epsilon)$  in Newtons per millimeter to the  $p$ th power ( $N/mm^p$ )

Additionally, the piano-hammer mass may be approximated across the keyboard by

$$m = 11.074 - 0.074n + 0.0001n^2$$

4

## Piano String Parameters

Piano string parameters for the medium-sized *Parlour* grand piano:<sup>4</sup>

Shortest string:

length  $L = 52$  mm

diameter  $d_1 = 0.775$  mm

### String Striking Point

1/8 to 1/24

across upper sixty notes

### String Tension $T$

$$T = (2fL)^2\mu = (2f)^2LM = \pi\rho_s(Lfd_1)^2$$

where

$f$  = note frequency  $\in [A_2, C_5] = [27.5, 4186]$  Hz

$\mu$  = linear mass density

$M$  = entire string mass

$\rho_s$  = density of steel core (7860 kg/m<sup>3</sup>)

<sup>4</sup>Grand piano manufacturing in Estonia: The problem of piano scaling. Proc. Estonian Acad. Sci. Engin., 1999, v.5, N.2 (co-authors J. Engelbrecht, A. Mägi), 155-167. (PDF:645k).

- Should strive for  $Lfd_1 = \text{constant}$
- In practice, the tension is distributed linearly or parabolically
- Uniform tension is good for the frame.
- Treble tension is pretty uniform at 620 N

### String Count

- One string per note for first ten notes ( $A_0$ – $F\#_1$ )
- Two strings per note for notes 11–25 ( $G_1$ – $A_2$ )
- Three strings per note for notes 26–88
- For a constant-tension break, given  $T_{26} = 620$  N, choose  $T_{25} = 760$  N (stepping from 3 to 2 strings)
- Recommended  $T_1 = 1320$  N and reduce gradually to  $T_{25}$ , e.g.,  $T_{11} = 840$ ,  $T_{10} = 1188$  N, with linear decrease from  $T_1$  to  $T_{10}$  and  $T_{11}$  to  $T_{25}$

### String Core Diameters

We have initial string tensions  $T_{0n}$ , lengths  $L_n$ , and frequencies  $f_n$  which imply the initial mass densities:

$$\mu_{0n} = (2f_n L_n)^{-2} T_{0n}, \quad n = 1, \dots, 88$$

Non-wound string diameters:

$$d_{1n} = \sqrt{\frac{4\mu_{0n}}{\pi\rho_s}}, \quad n = 1, \dots, 88$$

rounded to the nearest multiple of 0.025 mm, resulting in new linear mass densities

$$\mu_n = \frac{\pi}{4} \rho_s d_{1n}^2$$

and tensions

$$T_n = (2f_n L_n)^2 \mu_n$$

### Tensile Strength of Steel Wire Core

To accuracy 1.5%, we have

$$[\sigma] = 321.235(1 - 0.3982d_1 + 0.1033d_1^2) \quad \text{kg/mm}^2$$

where the core diameter  $d_1$  is given in mm.