MUS420 Lecture
Introduction to Physical Signal Models

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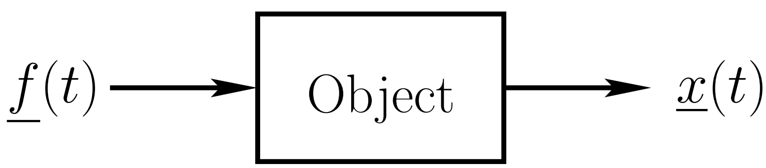
Outline

- Signal Models
- Physical Signal Models
  - Formulations
  - Simple Examples
  - Preview of Topics

What is a Model?

For our purposes, a *model* is any form of computation that *predicts* the state $\underline{x}(t)$ at some future time t of a physical object or phenomenon based on

1. the object's initial state $\underline{x}(0)$
2. any "input" (external) forces $\underline{f}(t)$ on the object



In our case, typically,

$$\begin{aligned} \underline{f}(t) &= \text{vector of } \textit{performance control} \text{ variables} \\ \text{Object} &= \textit{musical instrument} \text{ and/or } \textit{audio effects} \\ \underline{x}(t) &= \textit{object state} \end{aligned}$$

The *output sound* is some function of the object state $\underline{x}(t)$.

When to Model

- Physical models are desirable when *many dimensions of expressive control* are needed
- This need is best appreciated by *performing musicians*
- Audience members only hear the final music played, so they are not in a position to judge the quality of multidimensional control
- Skilled musicians normally use a *narrow control subspace* in any given performance

Signal Models

Popular *non-physical signal models:*

- Recordings (Samples)
- Spectral Models
- Virtual Analog (using classic sawtooth/square/etc. waveforms)

Recordings and spectral models can *emulate* physical models by storing a lot of data, indexed by control parameters.

Recordings (Samples)

A very simple signal model is a *recording* of the desired sound indexed by controller state (key-press, pedal, ...)

- Such a procedure is called instrument *sampling*
- For example, consider the following example *sampled piano:*
  - The *Synthogy Ivory* (a $349 software product in 2006), ships as 40 Gigabytes on ten DVDs (three sampled pianos)
  - Every key is sampled
  - 4–10 "velocity layers"
  - Separate recordings with the soft pedal down
  - Separate "release" recordings, for multiple striking velocities
- Sampling is highly laborious, but nevertheless the most common method
- Dimensions of expressive performance are limited
- Continuously-controlled instruments such as bowed-string, wind, and brass instruments are drastically oversimplified

  - Piano performance is well digitized by MIDI (key-number, key-velocity)
  - Piano *pedals* are *continuous* controls, however
  - Bowed-string and wind instruments have multiple continuous dimensions of control (many MIDI "controllers" needed in principle)
- *Interaction with the instrument state* is invariably sacrificed

The main advantage of sample-based sound synthesis is the high sound quality (limited only by the original recordings and subsequent signal processing)

Spectral Models

As discussed in Music 421 (next year), *spectral* models are inspired by the mechanics of *hearing*

- They are often based on the *Short-Time Fourier Transform* (STFT)
- There are also signal models, such as *Linear Predictive Coding* (LPC) whose success derives from how well they match spectral characteristics of hearing
- Frequency-Modulation (FM) synthesis is typically developed by tweaking FM parameters to match short-term audio spectra, so it can be regarded as a spectral modeling technique with extremely low computational requirements
- Other well known signal models rooted in the spectral point of view:
  - *phase vocoder*
  - *additive synthesis*
  - *spectral modeling synthesis* (sines + noise, sines + noise + transients, etc.)

Virtual Analog

Analog synthesizers, such as the modular Moog and ARP synthesizers, typically used elementary waveforms such as sawtooth, pulse train, and the like

- A variety of signal models has been developed for generating such waveforms digitally without incurring the aliasing associated with simply sampling the ideal waveforms
- A special issue of the IEEE Transactions on Audio, Speech, and Language Processing (May 2010) was devoted to this area
- The same issue also includes papers on more model-based approaches in which the original analog circuit is digitized using general methods, some of which will be discussed in later lectures
- *Waveform* signal models will *not* be discussed further in this course, so see the many references in the text for more on such methods

Physical Signal Models

We now turn to the main subject of this course, *physical models of musical instruments and audio effects:*

- Review physical models in general
- Summarize important formulations
- Indicate course emphasis relative to the "big picture"

All We Need is Newton

- Since there are no relativistic or quantum effects to worry about (yet) in musical instruments or audio effects, Newtonian mechanics will suffice for our purposes[1]
- Newton's three laws of motion can be summarized by the classic equation

$$f = m\,a \qquad \text{(Force = Mass times Acceleration).}$$

- Models based on Newton's laws can quickly become complex
- We will usually need many further simplifications that preserve both sound quality and expressivity of control

[1] Actually, quantum will be needed exactly once.

Physical Model Formulations

Below are names of various kinds of physical model representations we will consider:

- Ordinary Differential Equations (ODE)
- Partial Differential Equations (PDE)
- Difference Equations (DE)
- Finite Difference Schemes (FDS)
- (Physical) State Space Models
- Transfer Functions (between physical signals)
- Modal Representations (Parallel Second-Order Filters)
- Equivalent Circuits
- Impedance Networks
- Wave Digital Filters (WDF)
- Digital Waveguide (DW) Networks

History

2 SOUND SYNTHESIS AND PHYSICAL MODELING

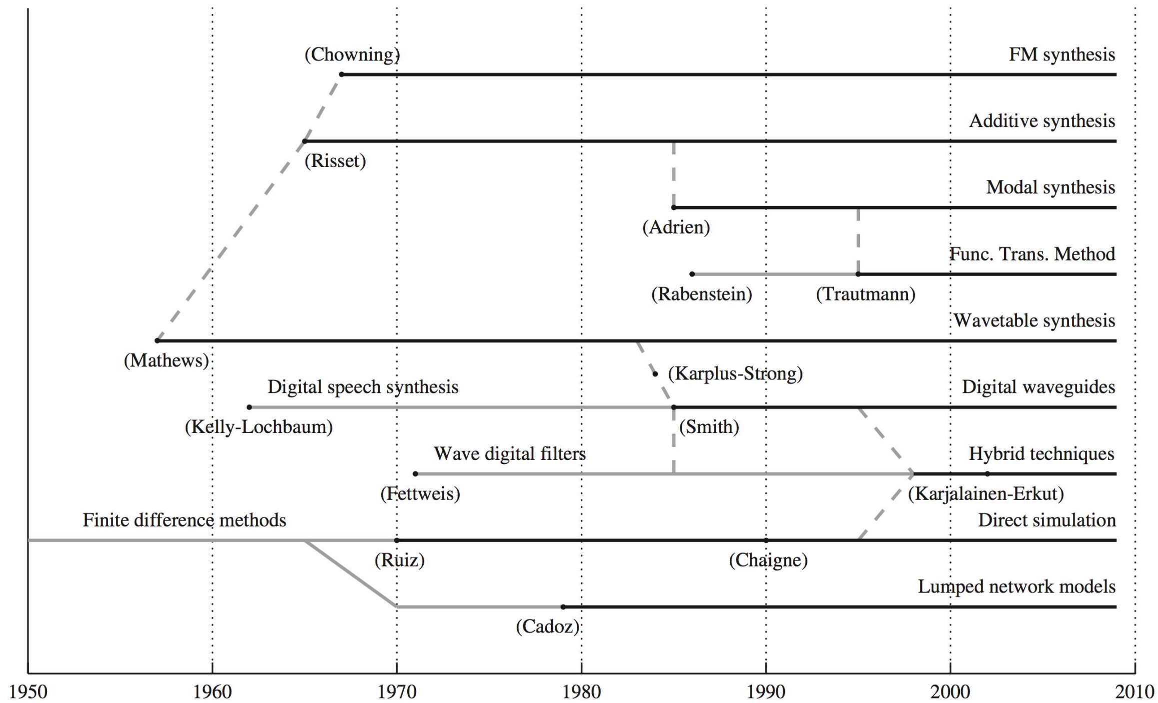


Figure from **Numerical Sound Synthesis**
Stefan Bilbao
Wiley 2009