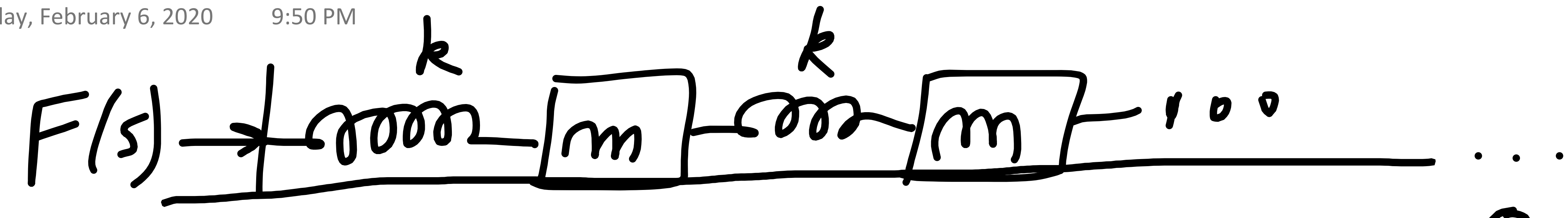
Mass Spring Chain

Thursday, February 6, 2020 9:50 PM



$$R(s) \triangleq \frac{F(s)}{V(s)} = \frac{k}{s} \,\|\, (ms + R(s)) \triangleq R_1 \,\|\, (R_2 + R)$$

$$= \frac{R_1 R_2 + R_1 R}{R_1 + R_2 + R} \Rightarrow \cancel{R R_1} + R R_2 + R^2 = R_1 R_2 + \cancel{R_1 R}$$

$$\Rightarrow R^2 + R_2 R - R_1 R_2 = 0$$

$$\Rightarrow R = -\frac{R_2}{2} \pm \sqrt{R_2^2 + 4 R_1 R_2} / 2$$

$$= -\frac{ms}{2} \pm \sqrt{\left(\frac{ms}{2}\right)^2 + mk}$$

$$= \frac{ms}{2}\left[-1 + \sqrt{1 + \frac{4\cancel{m}k}{m^{\cancel{2}} s^2}}\right]$$

$$\underset{\omega_0^2 \triangleq k/m}{=} \frac{ms}{2}\left[-1 + \sqrt{1 + \frac{4\omega_0^2}{s^2}}\right]$$

$$\xrightarrow{|s|^2 \ll \omega_0^2} \frac{m\cancel{s}}{\cancel{2}}\left[\cancel{2}\,\frac{\omega_0}{\cancel{s}}\right] = m\omega_0 = m\sqrt{\frac{k}{m}} = \sqrt{km}$$

$$\boxed{R(s) \simeq \sqrt{km} \qquad \omega \ll \omega_0 = \sqrt{k/m}}$$

"Wave Impedance"

Below cut-off frequency $\omega_0 = \sqrt{\frac{k}{m}}$, mass-spring-chain "looks like a <u>dashpot</u>"

<u>NO DYNAMICS</u>

All work performed <u>propagates</u>