

## Visualizing Harmony

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ICMC 2001  
Havana, Cuba

## Krumhansl-Schmuckler Algorithm

- Statistical measurement of the key
- Add durations of each pitch class
- Compare duration pattern to key prototype
- Choose best match as the key

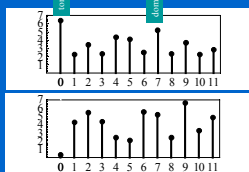
## Krumhansl-Schmuckler Algorithm

- Statistical measurement of the key
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Example:

Typical major  
key distribution  
of notes:

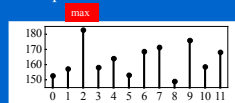
Example data  
measured from  
music:



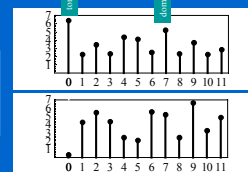
## Krumhansl-Schmuckler Algorithm

- Statistical measurement of the key
- Add durations of each pitch class
- Compare duration pattern to key prototype
- Choose best match as the key

Example:



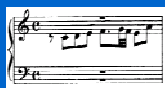
D Major is best fit



## Well-Tempered Clavier

```

!!!COM: Bach, Johann Sebastian
!!!VEN: The Well-Tempered Clavier, Volume 1, Fugue 1.
!!!SCT: BWV 846b
**kern **kern **kern **kern
*clefF4 *clefF4 *clefG2 *clefG2
*M4/4 *M4/4 *M4/4 *M4/4
=1 =1 =1 =1
1x 1x 1x 1x
- - 8c -
- - 8d -
- - 8e -
- - 8f -
- - 8g -
- - 8a -
- - 8b -
- - 8c -
=2 =2 =2 =2
    
```



Result of Humdrum **key** command:

Estimated key: C major (r=0.8640) confidence: 22.3%

## Well-Tempered Clavier (2)

Book 1:

Fugue	Key	r	confidence
Fugue 1:	C major	(r=0.8640)	confidence: 22.3%
Fugue 2:	C minor	(r=0.8688)	confidence: 32.1%
Fugue 3:	D major	(r=0.8681)	confidence: 58.2%
Fugue 4:	D minor	(r=0.8681)	confidence: 58.2%
Fugue 5:	E major	(r=0.8681)	confidence: 56.6%
Fugue 6:	E minor	(r=0.8681)	confidence: 56.6%
Fugue 7:	F major	(r=0.8681)	confidence: 56.6%
Fugue 8:	F minor	(r=0.8681)	confidence: 56.6%
Fugue 9:	G major	(r=0.8681)	confidence: 56.6%
Fugue 10:	G minor	(r=0.8681)	confidence: 56.6%
Fugue 11:	A major	(r=0.8681)	confidence: 56.6%
Fugue 12:	A minor	(r=0.8681)	confidence: 56.6%
Fugue 13:	B major	(r=0.8681)	confidence: 56.6%
Fugue 14:	B minor	(r=0.8681)	confidence: 56.6%
Fugue 15:	C major	(r=0.8681)	confidence: 56.6%
Fugue 16:	C minor	(r=0.8681)	confidence: 56.6%
Fugue 17:	D major	(r=0.8681)	confidence: 56.6%
Fugue 18:	D minor	(r=0.8681)	confidence: 56.6%
Fugue 19:	E major	(r=0.8681)	confidence: 56.6%
Fugue 20:	E minor	(r=0.8681)	confidence: 56.6%
Fugue 21:	F major	(r=0.8681)	confidence: 56.6%
Fugue 22:	F minor	(r=0.8681)	confidence: 56.6%
Fugue 23:	G major	(r=0.8681)	confidence: 56.6%
Fugue 24:	G minor	(r=0.8681)	confidence: 56.6%

## Well-Tempered Clavier (3)

Book 2:

Prelude 1: C major	( $r=0.9382$ )	confidence: 59.2%
Prelude 2: C minor	( $r=0.9380$ )	confidence: 70.1%
Prelude 3: C# major	( $r=0.9524$ )	confidence: 78.0%
Prelude 4: C# minor	( $r=0.7837$ )	confidence: 38.9%
Prelude 5: D major	( $r=0.8226$ )	confidence: 15.1%
Prelude 6: D minor	( $r=0.8729$ )	confidence: 56.5%
Prelude 7: E-flat major	( $r=0.8962$ )	confidence: 15.8%
Prelude 8: E-flat minor	( $r=0.7776$ )	confidence: 44.2%
Prelude 9: E major	( $r=0.8248$ )	confidence: 8.8%
Prelude 10: E minor	( $r=0.8239$ )	confidence: 45.4%
Prelude 11: F major	( $r=0.9037$ )	confidence: 39.9%
Prelude 12: F minor	( $r=0.7862$ )	confidence: 51.4%
Prelude 13: F# major	( $r=0.7834$ )	confidence: 15.6%
Prelude 14: F# minor	( $r=0.8283$ )	confidence: 52.1%
Prelude 15: G major	( $r=0.8531$ )	confidence: 18.5%
Prelude 16: G minor	( $r=0.7928$ )	confidence: 44.8%
Prelude 17: A-flat major	( $r=0.8958$ )	confidence: 33.5%
Prelude 18: A-flat minor	( $r=0.8473$ )	confidence: 62.1%
Prelude 19: A major	( $r=0.7928$ )	confidence: 1.8%
Prelude 20: A minor	( $r=0.8704$ )	confidence: 87.0%
Prelude 21: B-flat major	( $r=0.8459$ )	confidence: 44.7%
Prelude 22: B-flat minor	( $r=0.8329$ )	confidence: 57.5%
Prelude 23: B major	( $r=0.8621$ )	confidence: 16.3%
Prelude 24: B minor	( $r=0.7644$ )	confidence: 0.8%

## Window Size

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- How much music to consider when determining key?
  - 1 measure is usually not enough

One measure:



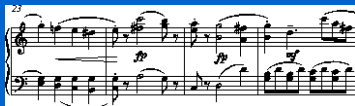
## Window Size

- Compositions are usually not all in one key
- How much music to consider when determining key?
  - 4 measures is often good
  - depends on meter, rhythm, etc.

One measure:



Four measures:



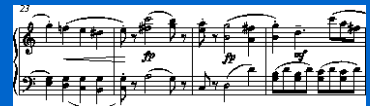
## Window Size

- Compositions are usually not all in one key
- How much music to consider when determining key?
  - 4 measures is often good
  - depends on meter, rhythm, etc.
- Safe way is to consider all window sizes for analysis

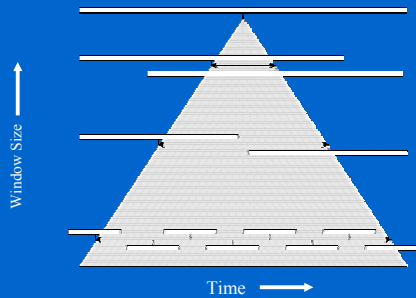
One measure:



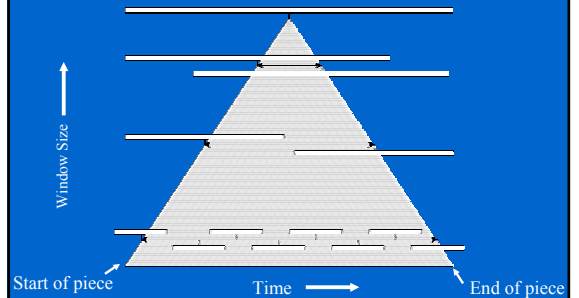
Four measures:



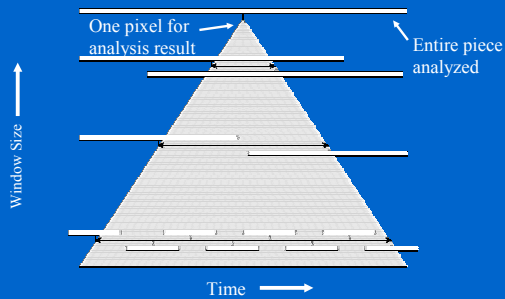
## 2D Plot for Key Analyses



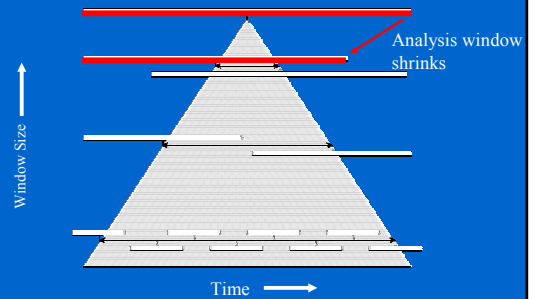
## 2D Plot for Key Analyses



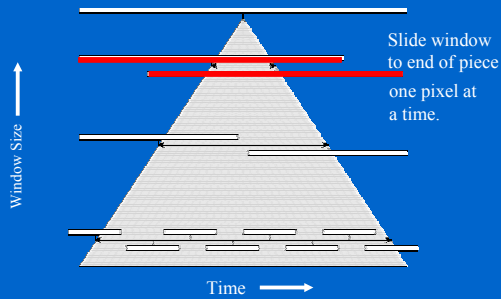
## 2D Plot for Key Analyses



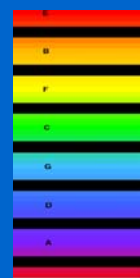
## 2D Plot for Key Analyses



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## Key to Color Mapping



- Rainbow mapped to circle of fifths
  - Mostly a diatonic mapping
  - Sufficient for tonal harmony
  - Any mapping is possible
  - Brightness/Contrast can also be used
- For example:  
Major/Minor => Bright/Dark

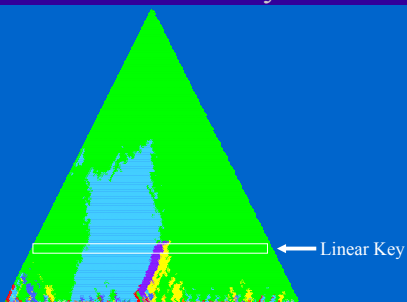
## Example: Mozart Sonatina

Key changes indicated in the score: C major, D minor, F major, G major.

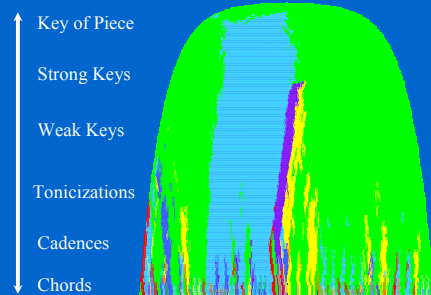
## Linear Picture of Key



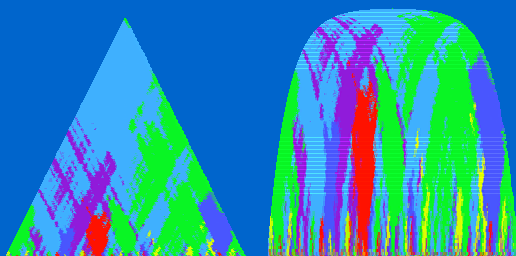
## 2D Picture of Key



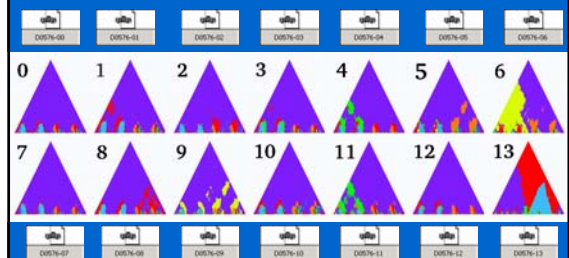
## Logarithmic Scaling



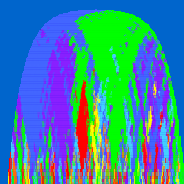
## WTC1 Fugue 3



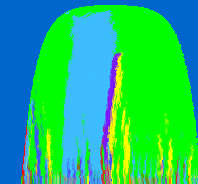
## Schubert Variations



## Atonal Music

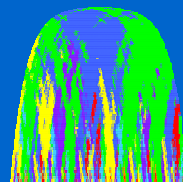


Webern Op. 27, Mvt. 1

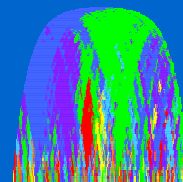


Mozart Sonatina in C, Mvt. 1

## Pre-Tonal Music



Petrus de Cruce  
Motet  
(13th century)

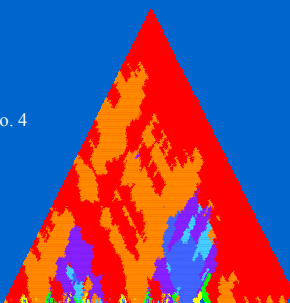


Webern Op. 27  
(20th century)

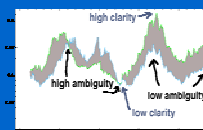
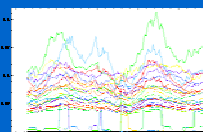
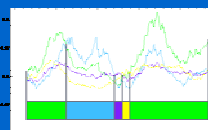
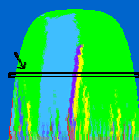
## Extended Tonality

Scriabin

Op. 11  
Prelude No. 4

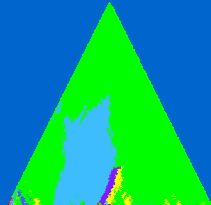
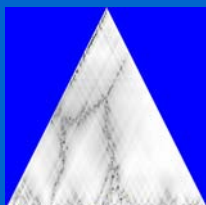


## Plot Variations



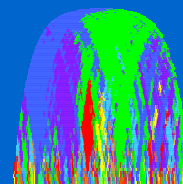
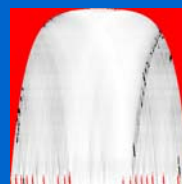
## Plot Variations

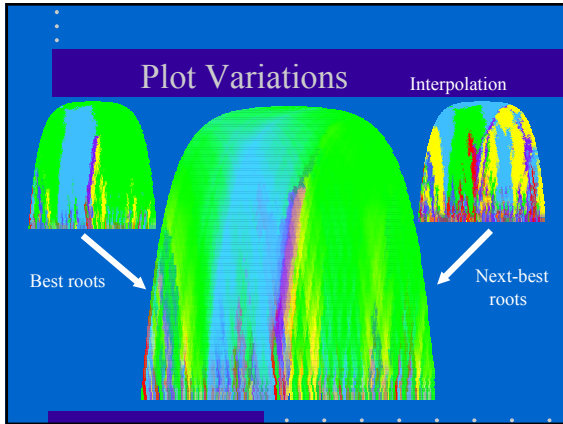
Ambiguity



## Plot Variations

Clarity





Further Information

- Keyscape website:  
<http://www-ccrma.stanford.edu/~craig/keyscape>
- Written description:  
Presented at ICMC 2001, Havana, Cuba  
<http://www-ccrma.stanford.edu/~craig/papers/01>