SCI220 – Foundations of Musical Acoustics
Cogswell Polytechnical College
Fall 2008

Week 6 – Class Notes

Oscillation Modes (FMA)

Chapter 6: The Modes of Oscillation of Simple and Composite Systems

6.1 Properties of Simple Oscillations

For a circular moving system (for example, the projected shadow of a nail mounted on a rotating turntable) the resulting sinusoidal motion has an amplitude equal to the radius of its path and whose frequency is equal to the number of revolutions a reference point (nail) makes per unit of time.

Regarding the forces that cause sinusoidal motion, we can say the following: if a body feels a leftward restoring force proportional to its displacement when it is to the right of its central position and a similar rightward force when it is to the left, then it will execute a sinusoidal motion when it is pulled aside and released. We can also notice that a displacement (d) is proportional to the magnitude of the restoring force.

For damped sinusoidal motion, the magnitude of the retarding force is proportional to the velocity of flow (in a fluid system). Meaning that when this dragging force is added to the restoring force, an undamped sinusoidal oscillation will become a damped oscillation.

The damped sinusoidal oscillation of an object of mass M, will move under the influence of two forces dependent of both the stiffness coefficient S, and on the damping coefficient D. This relationship can be written as:

$$f = \left(\sqrt{(S/M)}\right) \text{times a constant}$$

$$t_{1/2} = (M/D) \text{times a constant}$$

where f is the frequency of oscillation and $t_{1/2}$ is the halving time.

These formulas state that an oscillator will increase its characteristic frequency if a stiffer spring is attached to the given mass, or its halving time will be reduced if a more viscous damper is used. It can be noticed as well that a greater mass will reduce the vibration frequency and will lengthen its halving time is the stiffness and damping coefficients are unchanged.

6.2 Possible Oscillations of a Mass Supported by Springs

If we have a system of a steel nut suspended by two rubber bands, we can observe the following elemental vibration possibilities: transversal (side-to-side motion), longitudinal (back and forth motion along the axis created by the bands), and torsional (clockwise/counterclockwise twisting motion).

Exciting this system by means of striking the mass, will produce all of the possible oscillation possibilities and will produce a complex sound. It can be noticed as well that striking the mass in a up-down motion will only excite the oscillations occurring in this direction (transversal) and will fail to excite the rest. The same occurs for an on-axis strike.

6.3 Transverse Oscillations of Two Masses Connected by Springs

As seen on figure 6.7 (p.81) the first transverse vibrational mode of two mass-spring system, the two masses will move back and forth exactly in-phase with each other. If the amplitude of the motion is less than 10% of the longitudinal spacing of the masses, the oscillation will be described as a damped sinusoidal oscillation. Mode 2 of this system shows that the two masses always move in opposite directions. Both oscillate in a sinusoidal manner and have the same frequency.

Further examination of figure 6.7 suggests that the middle band is not nearly stretched as the other two in mode 1 as it is on mode 2. This means that at any given instant during the oscillation, the total restoring force exerted by the two rubber bands attached to a single mass will be smaller when the motion is of the type shown in mode 1 than in mode 2. The net stiffness coefficient is less in mode 1 than in mode 2, thus mode 1 is expected to oscillate at a lower frequency.

6.4 More Than Two Masses Connected by Springs

We can observe from the previous section that a one-mass oscillating system will have only one vibration mode, and that a two-mass will thus have 2 vibrational modes. We could expect that a three-mass system will have 3 vibrational modes and that a four-mass system will present 4 vibrational modes. This guess is actually correct.

It can be noticed as well that the first mode of all our systems are very similar in that it has one "hump" and that mode 2 has two, and mode 3 and 4 will have three and four of these "humps" respectively.

We could correctly conclude that a string will have all of these characteristic vibrational modes and possibly more since it can be thought that the string is made up of thousands of masses (molecules) arranged in a row. However, out of all of these vibrational possibilities, only the first 2-3 dozen have musical relevance.

Chapter 10: Sinusoidally Driven Oscillations

Repetitive excitation of an object can build up a very large amplitude of oscillation if the excitation frequency is approximately equal to the natural vibration frequency f the object.

10.1 Excitation of a Pendulum by a Repetitive Force

From the motor driven pendulum we can observe the following points:

1. At the beginning of the motion there is a random-appearing initial motion (transient) that eventually settles down to a steady sinusoidal oscillation.
2. This steady oscillation takes place at the driving frequency set by the motor, independent of the natural frequency of the pendulum.
3. The amplitude of this motion depends on the relationship between the driving frequency and the oscillating object. The maximum amplitude will be caused when the frequencies of both the driving force and the pendulum are exactly the same.
4. The amplitude of the pendulum's sinusoidal response will be weak if the driving frequency is much larger or smaller than the natural oscillation frequency. The response will be strong is the two frequencies are the same.

10.2 Properties of the Initial Transient Motion

Heavily damped oscillators do indeed settle down to their steady-state vibrations much more quickly than do lightly damped oscillators. The initial transient motion is the combination of a damped oscillation at the natural frequency of the pendulum plus the the steady oscillation that takes place at the driving frequency. This latter oscillation will persist after the transients die out.

10.3 The Influence of Variable Damping on the Steady Response

Half-amplitude bandwidth or resonance width $W_{1/2}$ is the range of driving frequencies within which the pendulum swings with a steady state amplitude for at least half the maximum amplitude attainable.

If the damping is increased enough to halve the amplitude of the maximum response, the corresponding bandwidth $W_{1/2}$ has been doubled, such that

$$W_{1/2} = \text{constant}(D)$$

Both the halving time $T_{1/2}$ and the maximum amplitude A_{max} for a sinusoidally driven oscillation are inversely proportional to the damping such that,

$$T_{1/2} = \text{constant}/D$$

$$A_{max} = \text{constant}/D$$

10.5 Steady Excitation of a System Having Two Characteristic Modes of Vibration

The motion of every mass of a sinusoidally driven complex system reaches maximum amplitude when the driving frequency is close to any one of the system's natural frequencies. In many cases this motion is out-of-phase with the driving force by a quarter cycle. At certain frequencies lying between those frequencies that are characteristic of the system's vibrational modes, the motion of all masses has a minimum amplitude. At these frequencies of minimum response, the masses again usually have a motion that is one-quarter cycle out-of-phase with the driving force.

At frequencies below the first oscillation mode of the system, the driven vibrations show the characteristic mode 1 shape despite the fact that there is a certain amount of mode 2 present. For frequencies above the 1^{st} mode of vibration, the vibrational shape will gradually change from that of mode 1 to the characteristic shape of mode 2. For frequencies higher than the natural frequency of mode 2, the shape of mode 2 will remain undisturbed.

10.7 The Transfer Response of a Tin Tray

The ability of the driver to excite a given mode depends on its position relative to the nodal lines and humps belonging to a mode's characteristic shape.

No matter what the frequency of the driving force is, all of the modes are excited to some extent, so that the degree of steady-state response observed at any given point depends on the summation of the responses of all of these modes, with proper account being taken not only of the fact that predominant contributors are those whose natural frequencies approximate the driving frequency, but also of the presence of varying amounts of lag between stimulus and response.

In places where two different modes give deflections of the same sign, so that the net disturbance is very large. Other places in which two different modes give equal and opposite disturbances, they will cancel each other out to give a negligible signal despite the fact that the overall disturbance in the plate is quite large. Finally, there are places in which only one mode contributes to the detected signal because the other mode is located along the nodal line of another mode.

10.8 Some Musical Implications

The exact placement of the soundboard resonance of a piano or harpsichord turns out to be much less critically involved in the musical nature of the instrument, but the average difference in frequency between adjacent modes is of considerable importance.

The transmission of sound from one point to another in a room is an example of the transfer response of a multi-resonance (3-dimensional) system.