SCI220 – Foundations of Musical Acoustics
Cogswell Polytechnical College
Fall 2008

Week 15 – Class Notes

Two Dimensional Surfaces (FMA)

Chapter 9: The Vibrations of Drumheads and Soundboards

9.1 Unraveling the Mode Shapes of a Glockenspiel Bar

Let us begin by remembering that the lowest frequency component in a set corresponds to the first characteristic vibrational mode of a bar. Then strength of excitation will be greater in the places where the vibration has its largest amplitude and the least at the nodal points. In the case of mode 1 the middle point of the bar as well as the end points will have a strong excitation. The nodes will be found at a quarter length from each end of the bar.

For mode 2, we can observe that the shape of the bar has an additional hump. In this mode we can find four regions of strong excitation along the bar. We can further induce that the vibrational shape of mode 3 will have three humps and that successive modes will have an additional hump with respect of the previous mode.

However, for mode 3 exciting the bar anywhere along its middle lines will fail to excite this mode and the only way we can excite this mode is by striking anywhere near the four corners. The resulting vibrational shape of mode 3 will be a side to side twisting shape.

We can summarize by sating that for any mode there are regions of strong vibrational disturbance (humps) separated by places in which there is no motion at all (nodes). A vibrational system always moves in opposite directions on the two sides of a node.

9.2 Mode Shapes of a Rectangular Plate Having Free Edges

A simple rectangular plate will behave much like a bar in its modes 1, 2, and 4. Shorter bars will have higher characteristic frequencies than do long bars. Each bar will be 70% of the length of its namesake an octave lower, thus halving the length of a bar will raise its frequencies about fourfold.

For rectangular plates we can describe two types of natural modes of vibration. One group has the humps arranged along the length of the plate, with nodal lines crossing it. The second group has the humps running crosswise on the plate, with longitudinal nodal lines.

There are however, modes that have characteristics of both groups. Meaning that the shape will be that of a twisting plate. The natural frequency belonging to any one of these complicated modes will always be higher than that of either of its “ancestors”.

For a single string we can indicate the characteristic shape by giving the number of the mode. In two-dimensional plates we can indicate the characteristic shape by naming both the longitudinal and transverse ancestors.

A sheet's twisting stiffness is enormously lower than is its bending stiffness, leading us to expect that the twisting mode will oscillate more slowly than the bending one.

9.3 The Effect of Various Boundaries

Let us describe the three major boundary conditions:

1. Free edge: has no externally applied constrains on the edge, thus it is free to move.
2. Clamped edge: the whole perimeter is clamped as in the jaws of a vise. The edge is fixed and it cannot move or tilt.
3. Hinged edge: the edges is hinged such that the plate does not move back and forth, but its tilting motion is not constrained.

The more bending there is in a mode shape, the higher the corresponding frequency. The additional bending found at the edges of a clamped plate produces higher frequencies of its various modes than those of its hinged-edge counterpart. However, the difference between the two cases is less for higher numbered modes than for the lower ones because the bending associated with the increasing number of humps will tend to drown the relatively constant amount of bending force by a clamped edge.

Too much stiffness imposed by wedging changes the plate modes from a sort that approximates hinged-edge modes to ones that are characteristic of boards having clamped edges.

9.4 Adjustment of Frequency Relations by Variations of Thickness

The perceived pitch of a composite sound depends on the frequency relations existing between its components, while the clarity of sound depends on these relations.

A flexible membrane can be seen as a clamped edge surface, while a circular plate can be represented as a hinged edges. Mechanically, the spring forces of a plate produced by the stiffness of the material try to restore the surface to its flat shape. The membrane, on the other hand, lacks stiffness so that it must be pulled tight at the boundary so that the tension can produce a restoring force.
The vibrational shape of a circular membrane under tension is exactly the same as those of a plate with hinged edges. Despite this similarity, the two sequences of characteristic frequency ratios are not alike. The plate's lower frequencies will be quite far apart from those found in a membrane.

The characteristic frequency of vibration given by $f=\sqrt{(S/M)}$ can help us understand what happens to a surface when we increase the mass M by adding wax to it in a maximum point of vibration. As a result the frequency will be lowered by increasing the mass. Adding a mass on a node will produce no frequency change at all.
Adding several tiny masses will cause a net shift of various frequencies which will be the aggregate of the changes produced by each lump acting separately.