SCI220 – Foundations of Musical Acoustics
Cogswell Polytechnical College
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Week 10 – Class Notes

Strings II (FMA)

Chapter 23: The Oscillations of a Bowed String

23.1 The Excitation Mechanism of a Bowed String

The important features of the bowed string excitation mechanism can be outlined by the following statements:

1. The sliding-friction behavior of a bow acting on a string can only sustain oscillations when the surface treatment (rosin) is such as to give a downward slope to the force-versus-bowing speed curve.
2. The steeply sloping portions of this curve correspond to the operating conditions in which the excitatory force is sensitively controlled by oscillatory variations in the string velocity *v* at the bowing point.
3. The shapes of the excitatory friction curves are such that the player can move the operating point for the oscillation toward a region of greater steepness either by pressing harder or by bowing more slowly.
4. The fact that the excitatory-friction characteristic curve is not straight (the slope varies from point to point along it) is an indication that heterodyne effects can occur, giving rise to regimes in which the oscillation is maintained by excitation taking place at several frequencies simultaneously. The bowing conditions which increase the steepness of the curve also increase its curvature.

23.2 The Resonance Curves and Regimes of Oscillation of a Bowed String

A response curve contains information on the frequencies ay which an applied sinusoidal force gives the maximum oscillatory velocity; the peaks on the response curve tell which frequencies best communicate with the bow friction in setting up regimes of oscillation.
The string will respond strongly to the driving force at the frequencies of each of these characteristic modes. The frequencies of these modes are in very nearly harmonic relationship. An excitation applied near the middle of a vibratory hump will produce much more of a response that will a driving force applied near a node.

A *regime of oscillation* is a state of the collective motion of an air column or string mode in which a nonlinear excitation mechanism (reed, bowing) collaborates with a set of air-column or string modes to maintain a steady oscillation containing several harmonically related frequency components, each with its own definite amplitude.

Experimental measurements show that the half-amplitude time for the decay of mode 1 of the A-string on a violin is about 0.5 seconds, about five times longer than the decay of modes 2 and 3, and about fourteen times longer than the decay times of modes 4 through 10. the damping rises very rapidly for the higher modes beyond mode 10. Since the heights of the resonance-curve peaks are closely related to the decay times of the corresponding string modes, the heights of the response peaks for any mode that are heavily damped will be reduced.

Pressing harder on the bow simply adds cooperative contributions from the other string modes, and the system plays in a regime of oscillation dominated by half a dozen peaks. When a string is shortened by pressing it against the fingerboard with the tip of a finger, the string-mode frequencies will of course be raised because of the shortened string, but at the same time the dampening of the modes will be increased by frictional effects at the fingertip. Meaning that the sound produced in this manner has a fundamental frequency higher than that of the natural sounding frequency of the string. Heavier bowing causes the pitch to drop down to the normal frequency as the main, lower-register regime takes over. This example is known by violinists as *harmonics*.

In harmonics the normal, low-register oscillation involving all of the modes is somehow disrupted so as to favor regimes based on peaks 2,4,6, etc (an octave higher), or on peaks 3,6,9, etc (giving a tone a twelfth higher). When a violinist wishes to play an octave harmonic, he fingers the string very lightly at the midpoint, which has a selective damping on the odd-numbered modes; this lowers the corresponding resonance peaks without altering the heights of the even-numbered ones.

23.3 The Effect of Inharmonicity and Damping on the Setting-Up of Regimes

A given resonance can participate to some extent in a regime of oscillation even when it is not perfectly aligned, provided that some harmonic of the generated tone lies reasonably well up the resonance peak.

1. In any multi-resonance oscillating system, a given resonance peak can take part in the regime only if its own natural frequency differs from that of the nearest harmonic of the tone by an amount that is less than the half-amplitude bandwidth $W_{1/2}$ of the peak.
2. Increasing the damping of a given mode of oscillation has two effects on the resonance-curve: a) the height of the peak is reduced, and b) the width is increased by the same factor. These two factors will have opposing effects on the ability of the resonance to participate in the regime of oscillation: 1) a reduction in the height of the peak means that the influence of this resonance diminishes, and 2) for a given small amount of detuning, and increase of the width means that the peak is given additional influence over the regime.
3. If a peak cannot be aligned perfectly, it must be assured that there is enough damping so that there is enough overlap of the peak with the closest sound component.

The resonance frequencies of a violin string in its normal environment are considerably closer to being harmonic than they are when the string is mounted on a rigid frame. The resonance widths are sufficiently broad that the peaks all find it easy to join in a regime of oscillation according to the requirements previously mentioned.

For example, a mezzo-forte bowed A4 sounds at a pitch that is about 5 cents higher than A-440 when the string is tuned in such a way as to place its plucked first-mode frequency at 440 Hz.

When an unbowed string is plucked a measurement of mode 1 shows that the initial, large amplitude vibration takes place at a frequency of oscillation that is noticeably higher than what is observed later on as the vibration dies away. The reason for this is that the large-amplitude vibration requires a slight stretching of the string to permit the existence of the vibrational hump. This stretching produces an increased average tension in the string, and the frequency-raising effect of the increased tension is only partially offset by the thinning of the string that is another consequence of the stretching. As the vigorous initial vibration dies away, the frequency shift due to the tension change dies away even more rapidly, so that we quickly arrive at the steady frequency that is characteristic of the small-amplitude vibration of the string's first mode.

When a violin string is bowed in a way that maintains a fairly large-amplitude oscillation, the resonance peak for mode 1 may shift upward in frequency by a dozen cents due to the vibratory increase in tension. The higher modes are not shifted upward so much, however, because of the non uniformity of this added tension along the string. Since the playing frequency is determined jointly by all of the resonance peaks that participate in the regime of oscillations, the 5-cent pitch rise associated with bowing is less than the 12-cent shift belonging to the string's first mode.

23.4 A Description of the Bowing Mechanism

The bowing of a violin string works in the following manner:
When the bow is placed on the string and drawn to the side, the string sticks to the bow, which pulls it aside until the elastic restoring force produced by the string tension becomes large enough to break the string loose from the bow. It now swings back in much the same way it would after slipping off the plectrum of a harpsichord jack; there is a small amount of damping produced by the rapid (low friction) sliding of the string against the steadily moving bow hair. At the end of its backward swing the string will come to rest and then recommence its motion in the direction of the bow velocity. At this time it is once again caught by the large sticking friction of the bow and carried forward to begin a new cycle of the oscillation.

In a vibratory pattern seen at the bowing point of a string, the longer more gently sloping part of the oscilloscope trace shows the steady upward motion of the string as it is carried along the bow. The duration of this part of the cycle is known as the *sticking time.* When the string reaches the upper limit of its travel, it breaks away from bow and runs downward quickly to the opposite extreme of its motion, where is re-caught by the bow for a steady upward trip. This is called the *flyback time.* The ratio of the flyback time to the total repetition time will be equal to the ratio of the bowing point distance B to the total string length L.

The vibration recipe observed at the bowing point is the same as the recipe for the amplitudes of the modes of a plucked string. The expected effects of large bow-hair width on a stiff string would be similar to those of a broad plectrum exciting it.

At any frequency component having a node at the bowing point is expected to be missing unless the bow has an appropriate width.

If one wishes to play loudly, it is clear that at any point on the string must make a wider excursion to each side of center in the course of each oscillation. Since the number of these back-and-forth trips per second is fixed by the playing frequency, we are led to conclude that the point must move with greater velocity to cover a larger round trip distance in the time of each oscillation. Because the bow and the string are moving together during one part of each cycle and because the string cannot move faster than the bow, it is obvious that loud playing demands fast bow motion.

Next we must consider the limitations of bow pressure in order to control the string vibrations adequately. The minimum pressure is that which is sufficient to carry the string along with the bow. The bow must be able to synchronize all the string modes into a motion of the desired sawtooth type. If some string modes are somewhat inharmonic or if their damping is high, more bow pressure will be required. However, the bow pressure must be small enough to allow the string to break loose cleanly at the end of its sawtooth swinging motion in order to make a good flyback.

On the other hand, if the pressure is too high between the moving bow and the string, the string simply pulls to one side, scraping, and stuttering against the bow hair without ever going into oscillation.

When the string sticks to the bow and is carried forward with it, and impulse is sent along the string toward its fixed end. This impulse is reflected back at the fixed end and comes back in its inverted form to the bowing point. If the bow pressure is not excessive, the impulse succeeds in breaking the string free in a manner that is quite reminiscent of the way in which the reflected pulse from a piano hammer blow returns to throw the hammer off the string.

The nearer the bow is to the bridge, the narrower is the range within which the player must maintain the pressure it exerts.

23.5 The Bridge Driving Force Spectrum

The sideways force exerted by a string on its anchorage at any instant during its vibration depends not only on the tension under which the string is kept, but also on the angle to which the string end is momentarily deflected.

In order to estimate the magnitude of the driving force F_n exerted n the bridge by the nth vibrational mode of a string, one must multiply the the amplitude A_n of the mode by its serial number n and by the tension T of the string, such that:

$$F_n = nTA_n \text{ x (a constant)}$$

When bowing at a node (nodal driving point), there is a small motion so that the bow can provide some excitation to the corresponding mode.

Chapter 24: Instruments of the Violin Family

24.1 The Body and the Bridge of Instruments of the Violin Family

It is customary to think about instruments of the violin family as being made up of three distinct parts: 1) the sound-generating portion of the instrument, consisting of the bow and the strings working cooperatively; 2) the body, whose resonances strongly influence the way the sound is radiated into the room; and 3) the bridge, which mediates between the oscillating strings and the body.

Each of the violin-family instruments consists of two arched wooden plates (top and back) joined at their perimeters by thin strips of wood called ribs. The foot of the bridge on the side carrying the treble strings is supported by a *soundpost* that is lightly wedged between the top and back plates; its placement serves both a structural supportive role as well as coupling the vibrations of one plate directly to the other. Under the bridge foot of the bass side a long strip of wood known as the *bass bar* is glued onto the inner surface of the top plate, running more or less parallel to the direction of the strings. The bass bar also couples the bridge vibrations effectively to both rounded portions of the top plate. The two *f-holes* influence the vibration properties of the top plate in a direct way and serve as passageways for the air enclosed in the body can communicate its oscillations to the room as part of the total radiation process.

The bowed string has two very different ways of exerting a driving force on the bridge. The most obvious one comes directly from the side-to-side oscillation of the string in a direction parallel to the motion of the bow. This excitatory force is referred to as the *direct excitation* of the bridge. A lever like action of the bridge is required to convert it into a force at right angles to the plate surface that can effectively drive the body of the instrument.

The second driving force relies on the fact that the string has a great deal of downbearing means that oscillatory changes in string tension that give rise to corresponding changes in the downward force exerted by the string on the bridge, a force which is ultimately applied to the top plate. This *indirect excitation* takes place at twice the vibration frequency of the string.

The direct driving force F_n^{dir} produced by the corresponding string acting alone has an amplitude that is proportional to the vibrating amplitude A_n of that mode. The indirect excitation of a single string mode excites an amplitude An to the corresponding indirect bridge-force F_n^{ind} is proportional to the square of An.

The *main air resonance* of the instrument is a consequence of the resonant excitation of the lowest characteristic mode of vibration of the air within the violin body. The air within a violin body acts in exactly similar fashion as a spring upon which the mass of air in the f-holes can oscillate. The natural frequency of such bottle-shaped air resonator will be lowered if the volume of enclosed air is increased, and it will be raised if the area of the f-holes is increased.

The rocking of the bridge on its soundpost at low frequency alternately contracts and expands the volume of air contained within the body, so that the air is alternately exhaled and inhaled by the f-holes. This indicates that the f-holes themselves are able to function as a simple acoustic source. However, not every transfer of air through the f-holes will give rise to sound.

Main wood resonances are strong output peaks for string excitations taking place at frequencies near 440Hz. These resonances are traced to the vibrational mode of the wood body by itself. A “breathing mode” is one in which the body as a whole expands and contracts its total volume.

24.2 High-Frequency Radiation Properties of Bowed String Instruments

As the excitation frequency applied to the body by the strings rises, it excites the plates into increasingly complicated vibration modes, each one having more nodal lines than the one before.

Some energy loss within the wood itself is produced by the damping effects of cross-grain and along-the-grain frictional losses that rise sharply above 3500 Hz. Beyond this frequency, the string excitation is spent as frictional heating within the instrument.
Measured damping of the string modes correlate with the ability of the bridge to steal the vibratory energy of the string by passing it along to the violin body and the to the air.

24.3 Characteristic Features of the Violin, Viola, and Cello

Two of these features are resonance peaks: 1) the strong resonance associated with the lowest mode of oscillation of the air enclosed within the body of the instrument and 2) the equally strong resonance associated with the simplest of the vibrations of the body's wooden parts. The third feature is the broadly rising amplitude of the higher partials up to a frequency that can be predicted from a knowledge of the first-mode resonance of the bridge itself.

The violin has its four strings tuned in fifths to the notes G3, D4, A4, and E5, and has an air resonance near 290 Hz (close to D4). The wood resonance (product of a body resonance and mode 2 of the air within it) is located around 440 Hz.

The viola has its strings tuned a fifth below the violin at C3, G3, D4, and A4. The first mode air resonance lies around 230 Hz (near Bb3) and the main wood resonance is found near 350 Hz (near F4).

The string tunings and playing range of the viola are transposed a fifth below that of the violin, and the high-frequency behavior is also transposed by the same amount. However, an important difference lies in the lower two resonances that are not transposed down by a fifth. This relationship gives the viola a musical character distinctly different from the violin.

The cello has its strings tuned an octave below those of a viola (a 12th below a violin) at C2, G2, D3, and A3. The main resonance is found around 125 Hz (between B2 and C3) and the main wood resonance lies near 175 Hz (about F3).

Bowed instruments of the violin family were perfected during the 17th and 18th centuries, giving us the modern violin, viola, and cello. The lowest member of the bowed string family today, the bass viol, is a descendant of the acoustically family of viols, which exist today as antique pieces. Early violin instrument makers were not successful in developing a complete set of instruments having overlapping playing ranges spaced apart in fifths or fourths.

24.5 Musical Properties of Bowed String Instruments

On an average basis, the violin radiates its low-frequency partials equally in all directions; its higher components are radiated in a progressively tighter beam in a direction perpendicular to the plates. However, superposed on this average behavior are the elaborate directional patterns of the separate partials mentioned above. It is a complicated radiation pattern for each partial of a violin tone (pattern that changes drastically for any frequency change) that distinguishes the violin family from other instruments.

String motion at the bowing point shows small fluctuations in the oscillation. Measurements of the separated fundamental and second-harmonic components of a violin tone showed the variations to be essentially random and spread over a frequency range of less than 1% (or about 20 cents). This fact means that there is a diffuseness to the string tone which has enormous implications to the musician. It allows for larger tuning errors to be made in an ensemble before the discrepancies become unacceptable and on the other hand, it permits the composer to write a wide variety of chords having many degrees of consonance and dissonance.