DAT330 – Principles of Digital Audio
Cogswell Polytechnical College
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Week 14 – Class Notes

Digital Signal Processing

Fundamentals of Digital Signal Processing

Digital signal processing is used to generate, analyze, or otherwise manipulate signals in the digital domain. It employs sampling and quantization, however it is only a processing method. A DSP system only processes signals (time-based sequence in which the ordering f values is critical). Digital audio signals only make sense, and can be properly processed if the sequence is preserved. Thus, DSP is a general application of data processing that uses mathematical formulas, or algorithms, to change the numerical values in the bitstream signal.

A signal can be any natural or artificial phenomenon that varies as a function of an independent variable. For example, when the variable is time, then changes in barometric pressure, temperature, current, or voltage can be considered signals that can be recorded, transmitted, or manipulated. Signal representations can be either analog or digital in nature, and both offer advantages and disadvantages.

Digital processing of acquired waveforms offer the following advantages over continuous-time signals: the use of components with lower tolerances, predetermined accuracy, identically reproducible circuits, a theoretically unlimited number of successive operations on a sample, and a reduced sensitivity to external effects.

Digital integrated circuits are compact, low in cost, and capable of complex processing (some digital operations are difficult or impossible in the analog realm). Examples include linear phase filters, adaptive systems, image processing, error correction, data reduction/compression, and signal transformation.

On the other hand, DSP has the following disadvantages: digital circuits always require power (not passive), cannot be used for very high frequency signals, large bandwidth requirements, digital technology is expensive to develop, and fast circuitry is needed. When used with analog applications, AD/DA conversion is required. The processing of very weak or strong signals requires proper treatment.

DSP Applications

In the 1960's, signal processing relied on analog methods to process signals in the continuous-time domain. Digital computers then where incapable to provide the means for signal processing. In 1965, the invention of the fast Fourier Transform to implement the discrete Fourier Transform, and the advent of powerful computers, inspired the development of theoretical discrete-time mathematics, and thus modern DSP.

Early applications of DSP where soil analysis in the exploration for oil and gas, radio and radar astronomy, telecommunications (in particular telephony), image processing in medical, astronomical, and television applications. Analytical instrumentation and industrial process control.

DSP represents rich possibilities for speech applications.

Discrete Systems

Digital audio signal processing is concerned with the manipulation of audio samples. Samples are represented as numbers.

When the independent variable, such as time (*t*), is continuously variable, the signal is defined at every real value of (*t*); thus, the signal is continuous. When the signal is only described at discrete values in time (*nT*), the signal is a discrete time signal. Although general discrete time signals and digital signals both consist of samples, a general discrete time signal can take any real value but a digital signal can only take a finite number of values. In digital audio, this requires approximation or quantization.

Linearity and Time-invariance

input output

$$x(n) \rightarrow y(n)$$

Linearity:

$$ax(n) \rightarrow ay(n)$$

$$x1(n) + x2(n) \rightarrow y1(n) + y2(n)$$

Time-invariance:

$$x(n-k) \rightarrow y(n-k)$$

Impulse Response and Convolution

The impulse response *h(n)* gives the full description of a linear time-invariant discrete system in the time domain.

Convolution is the scaling and overlapping samples of a filter response and an input to yield a new signal.

$$y(n) = \sum_{k=-\infty}^{\infty} x(k)h(n-k) = \sum_{k=-\infty}^{\infty} x(n-k)h(k)$$

$$y(n) = x(n)*h(n) = h(n)*x(n)$$

Complex Numbers

$$z = x + jy$$

Mathematical Transforms

A transform is a mathematical tool used to move between the time and frequency domains. Analog signal domains are time and frequency, while for sampled signals the two domains are discrete time and discrete frequency. Continuous transforms are used for signals in continuous time and frequency. Series transforms are used for continuous time and discrete frequency signals. Discrete transforms are applied to discrete time and discrete frequency.

For continuous signals, the Laplace transform is used to analyze continuous time and frequency signals. It maps a time domain function *x(t)* into a frequency domain, complex frequency function *X(s)*.

$$X(s) = \int_{-\infty}^{\infty} x(t) e^{-st} dt$$

The inverse Laplace transform performs the reverse mapping.

The Fourier transform is a special case of the Laplace transform which maps a time domain function *x(t)* into a frequency domain function *X(jω),* where *X(jω)* describes the spectrum (frequency response) of the signal *x(t)*.

$$X(j\omega) = \int_{-\infty}^{\infty} x(t) e^{(-j\omega t)} dt$$

The Fourier series is a special case of the Fourier transform and results when a signal contains only discrete frequencies and the signal is periodic in the time domain.

The discrete Fourier transform (DFT) is applied for discrete signals. For a signal of *N* samples, the *N*-point DFT transform is:

$$X(m) = \sum_{n=0}^{N-1} x(n) e^{(-j2\pi mn/N)}$$

The term *X(m)* is called the *m* bin, and describes the amplitude of the frequencies in signal *x(n)*, computed at *N* equally spaced frequencies. The *m = 0*, or bin 0 term describes the dc content of the signal, and all other frequencies are all harmonically related to the fundamental frequency corresponding to *m = 1,* or bin 1. Bin numbers specify the harmonics that comprise the signal, and the amplitude of each bin describes the power spectrum. There are both negative and positive frequencies, although only the positive half is shown.

The fast Fourier transform (FFT) is a set of algorithms used for fast and efficient spectral computations, taking advantage of computational symmetries and redundancies of the DFT.

The *z*-transform operates on discrete signals with complex values.

$$X(z) = \sum_{n=\infty}^{\infty} x(n) z^{-n}$$

Where *z* is a complex variable and z^{-1} represents a unit delay element.

Unit Circle and Region of Convergence

The Fourier transform of a discrete signal corresponds to the *z*-transform on the unit circle in the *z*-plane. The equation $z = e^{j\omega}$ defines the unit circle in the complex plane. The evaluation of the *z*-transform along the unit circle yields the functions frequency response.

For the set of finite complex variables *z* and *X(z)*, they are said to be valid if they in the region of convergence inside the unit circle. If these values fall outside of the region of convergence, they are said to be invalid. Thus, poles must be placed inside the unit circle on the the *z*-plane for proper stability, otherwise it is said that the system is instable.

Mapping from the *s*-plane to the *z*-plane is an important process since it can theoretically allow an analog transfer function to be "converted" into a discrete transfer function. The use of constants avoid instabilities in the resulting digital filters. Frequency warping effects are nonlinear relationships between analog and digital break frequencies.

Poles and Zeros

The transfer function *H(z)* of a LTD filter is defined to be the *z*-transform of the impulse response *h(n).* The spectrum of a function is equal to the *z*-transform evaluated on the unit circle. Because the transfer function of a digital filter can be expressed by its *z*-transform, it can be analyzed in terms of its poles and zeros. The zeros are the roots of the numerator's polynomial of the filter's transfer function, and the poles are denominator's roots. Generally, if we consider a flat spectrum along a flat *z*-plane, the magnitude of the transfer function will show a peak on top of a pole and a valley centered on a zero. The frequency response can be traced by this pattern along the unit circle. For example, the gain of a filter at any frequency can be measured by the magnitude of the contour, while the phase shift at any frequency is the angle of the complex number that represents the system's response at that frequency. Poles are represented by (x) and zeros as (o).

Observations:

1. Zeros are created by summing input samples, and poles are created by feedback.
2. A filter's order equals the number of poles or zeros it exhibits, whichever is greater.
3. A filter is stable only if all of its poles are inside the unit circle. Zeros can lie anywhere.
4. If all zeros lie inside the unit circle, the system is called a minimum-phase network.
5. If all poles are inside the unit circle and all zeros are outside, and if poles and zeros are always reflections of one another in the unit circle, the system is a constant amplitude or all-pass network.
6. If a system has zeros only, except for the origin, and they are reflected in pairs, the system is phase linear.
7. No real function can have more zeros than poles, and if the coefficients are real, poles are zeros occur in complex conjugate pairs (the plot is symmetrical across the real *z*-axis).
8. The closer the location of a pole and zero to the unit circle, the greater the effect on the frequency response.

DSP Elements

Although DSP applications require sophisticated hardware and software, DSP processing itself can be considered in three simple processing operations: summing, multiplication, and time delay.

Summing adds multiple digital values to produce a single result. And is expressed by:

$$y(n) = x1(n) + x2(n)$$

Multiplication changes the gain of a sample value by multiplying it with a coefficient. Expressed by:

$$y(n) = gx(n)$$

Time delay stores a digital value for one sample period *T*. The delay element (realized with shift registers or memory locations) is alternatively notated as z^{-1} in the *z*-domain. Delays can be cascaded. The time of a delay can be obtained by *nT,* where *T* is the sampling interval and *n* is the number of samples.

$$y(n) = x(n-1)$$

$$z^{-1}x(n) = x(n-1)$$

$$y(n) = x(n-1) + x(n-2) + \ldots + x(n-m)$$

In practice, these elemental operations are performed many times for each sample, in specific configurations depending on the desired result. In this light, algorithms can be devised to perform operations useful to audio processing, such as reverberation, equalization, data reduction, compression, and noise removal. For real-time operation, all processing for each sample must be completed within each sample period.

Digital Filters

Filtering shapes a signal's frequency response and phase, as described by linear time-invariant differential equations. Digital filters process each sample through a transfer function to affect the change in frequency response or phase. Digital filters can be designed from analog filters, making use of transformations to convert the characteristics of an analog filter to a digital filter. A digital filter can be represented by a general difference equation:

$$y(n) + b_1 y(n-1) + b_2 y(n-2) + \ldots + b_N y(n-N) = a_0 x(n) + a_1 x(n-1) + a_2 x(n-2) + \ldots + a_M x(n-M)$$

or can be expressed as:

$$y(n) = \sum_{i=0}^{M} a_i x(n-i) - \sum_{i=1}^{N} b_i y(n-i)$$

To implement the digital filter, the *z*-transform is applied to the difference equation, such that:

$$Y(z) = \sum_{i=0}^{M} a_i z^{-i} X(z) - \sum_{i=1}^{N} b_i z^{-i} Y(z)$$

The transfer function is written as:

$$H(z) = \frac{Y(z)}{X(z)} = \frac{\sum_{i=0}^{M} a_i z^{-i}}{1 + \sum_{i=1}^{N} b_i z^{-i}}$$

The transfer function can be used to identify the filter's poles and zeros. Zeros constitute feedforward paths and poles constitute feedback paths. By tracing the contour along the unit circle, the frequency response of the filter can be determined.

FIR Filters

Consider the general difference equation:

$$y(n) = \sum_{i=0}^{M} a_i x(n-i)$$

and the transfer function:

$$H(z) = \sum_{i=0}^{M} a_i z^{-i}$$

The equation has no poles, thus no feedback elements, resulting in a nonrecursive filter of the form:

$$y(n) = a_0 x(n) + a_1 x(n-1) + a_2 x(n-2) + \ldots + a_M x(n-M)$$

Any filter operating on a finite number of samples is known as a finite impulse response (FIR) filter. Thus, the filter has a finite duration and can only have zeros outside the origin, and can have linear phase (symmetrical impulse response). FIR filters respond to an impulse once, and are always stable. A nonrecursive structure is always an FIR filter, but an FIR filter does not always use nonrecursive structures.

FIR filters can be constructed as multi-tapped digital filter, functioning as building blocks for more sophisticated designs. To achieve a given frequency response, the impulse response of the coefficients of the FIR filter must be calculated. Simply truncating the extreme ends of the impulse response to obtain coefficients would result in an aperture effect and Gibbs phenomenon; the response will peak just below the cutoff frequency and ripples will appear in the passband and stopband. For example, the Fourier transform of an infinite (ideal) impulse response creates a rectangular pulse, a finite (real-world) impulse response creates a function exhibiting Gibbs phenomenon.

The choice of coefficients determines the phase linearity of the resulting filter. Linear phase is important for many audio applications because the filter's constant delay versus frequency linearizes the phase response and results in a symmetrical output response.

IIR Filters

The general difference equation contains *y(n)* components that contribute to the output value; these are feedback elements that are delayed by a unit of time *I*, and describe a recursive filter. These feedback elements are described in the denominator of the transfer function; because these roots cause *H(z)* to be undefined, certain feedback could cause the filter to be unstable. The poles contribute to an exponential sequence to each pole's impulse response; when the output is fed back to the input, the output in theory will never reach zero; this allows the impulse to be infinite in duration. This filter is known as an infinite impulse response (IIR) filter. In general, an IIR filter can be described as:

$$y(n) = ax(n) + by(n-1)$$

An IIR filter can have both poles and zeros, can introduce phase shift, and can be unstable if one or more poles lie on or outside the unit circle. IIR filters cannot achieve linear phase except when the poles in the transfer function lie on the unit circle. An IIR filter always has a recursive structure, but filters with recursive elements are not always IIR filters. Any feedback loop must contain a delay element; otherwise, the value of a sample would have to be known before it is calculated-an impossibility.

Filter Applications

Coefficients determine the filter's response and with appropriate selection of coefficients, highpass, lowpass, bandpass, and shelving filters can be obtained. A digital audio processor might have several of these sections at its disposal. By providing a number of presets, users can easily select the frequency response, bandwidth, and phase response of a filter. In this respect, digital filters are more flexible than analog filters. However, digital filters require considerable computational power, especially when new coefficients must be calculated at a relatively fast rate.

Specialized DSP Applications

By using the basic DSP building blocks it is possible to create sophisticated algorithms such as chorusing, phasing, reverberation, mixing, and noise reduction.

Digital Delay

When a delay time is small (< 10 ms), the frequency response of the signal is altered (comb filter); when the delay is longer (10-50 ms), an echo occurs.

A comb filter response has peaks and dips equally spaced through the frequency response, from 0 Hz to the Nyquist frequency. The number of peaks depends on the delay time; the longer the delay, the more peaks.

If the delay time of the circuit is slowly varied (0-10 ms), the time-varying comb filter creates a flanging effect. If the delay time is varied between 10-25 ms, a doubling effect is achieved. A chorus effect is provided when the signal is directed through several such blocks, with different delay variations. Comb filters can be either recursive or nonrecursive.

All-pass filters have flat frequency responses from 0 Hz to the Nyquist frequency; however, its phase response causes different frequencies to be delayed by different amounts. Phasing effects can be created with all-pass filters.

Digital Reverberation

Acoustical sound reflections are characterized by relative loudness (reflection from wall) and delay (size of room and frequency response). Because reverberation comprises a large number of physical sound paths, digital reverberation must similarly process many data elements.

With reverberation, the signal is mixed repetitively with itself during storage at continually shorter intervals and decreasing amplitudes. The stored information, from the delay lines, must be read out a number of time, and multiplied by factors less than unity. The result is added together to produce the effect of superposition of reflections with decreasing intensity.

The reverberation process can be represented as a feedback system with delay unit, multiplier, and summer.

Digital Mixing Consoles

A digital mixing console must accomplish the following tasks:

1. Gain control – digital preamplifier (potentiometer) and A/D converter to convert the analog position information of the variable resistor into digital form, and a multiplier to adjust the value of the digital audio data.
2. Mixing – requires a multiplexer and accumulator.
3. Signal selection/routing – requires a demultiplexer and encoding circuit to read the desired selection.

Any console must also provide equalization (LPF, HPF, shelving filters) by using biquadratic filter sections.

Loudspeaker Correction

Loudspeakers have nonuniform frequency responses, limited dynamic range, frequency-dependent selectivity, and phase nonlinearities that can degrade the reproduced audio signal. In addition, the listening room reinforces and cancels selected frequencies in different room locations, and contributes surface reflections, superimposing its own sonic characteristics on that of the audio signal.

DSP processing can correct frequency response and phase nonlinearities by optimizing the DSP programs coefficients for each loudspeaker. Comb-filter effects created by the acoustic paths between the direct and reflected sound can also be addressed with DSP.

Adaptive loudspeaker/room correction systems analyze a room's acoustical characteristics and from the collected data compensates for its own deficiencies, as well as those from the room.

Noise Removal

DSP can be used to improve the sound quality of a previously recorded material or restore a signal to a previous state. Tasks such as noise reduction/removal (click, hiss, hum, surface noise) are possible with DSP. In addition, signal loss or dropouts can be accurately synthesized.

Noise removal is divided into two tasks: 1) detection and 2) elimination of noise.

Frequency and amplitude analysis are done to find the unwanted artifacts (click, noise). Based on the resulting analysis, particular actions can be taken. For example, clicks are transient in nature, and uncorrelated to the audio signal. Because of this they can be easily detected from the wanted signal and can be corrected by de-clicking algorithms that interpolate new values over the detected defect.

With background noise removal, frequency analysis is done to determine the nature of the noise. Once a “fingerprint” of the noise is taken, it can be used to remove the noise from the signal.