Virtual Acoustic Tube Lab

RealSimPLE Project*
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Abstract

This laboratory exercise, geared toward a high school science audience, steps students through a number of experiments with a virtual acoustic tube implementation.

Contents

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1 Introduction

In this laboratory, you will experiment with a virtual acoustic tube model, as an introduction to the physics governing real acoustic tubes. As discussed in the digital waveguide model laboratory assignment,[1] virtual models allow engineers and researchers to experiment with theoretical modifications to various structures, and predict the acoustic consequences of these modifications through computer-based simulation. The acoustic tube, which we will simulate in this lab, is an important building block in musical acoustics.

Ideally, this lab should be preceded by exposure to the traveling waves laboratory assignment.[2]

2 Summary of Objectives

1. To learn about concepts related to pressure and volume velocity in an acoustic tube.
2. To understand the influence of various quantities on the adiabatic propagation of sound in fluids such as air.
3. To learn the concept of wave impedance and its effect on the propagation of longitudinal waves in an acoustic tube.
4. To understand wave reflection at a radius mismatch in a pair of acoustic tubes.
5. To experiment with a pair of virtual acoustic tube models, allowing for the viewing of responses to specified stimuli. While both models allow for user experimentation with air temperature, the second allows for a experimentation with parameters affecting wave reflection.

3 Background and Theory

The waves in an acoustic tube are longitudinal waves—that is, the direction of the fluctuating displacements that carry the wave is parallel to the direction of wave propagation. An animation[3] of longitudinal waves (and other types of waves) may be seen on the Web.

3.1 Speed of Sound

The speed of sound is the same in an acoustic tube as in the open air. In this section, we will explain some formulae relating to this quantity.

When sound propagates in an acoustic tube, there are two key quantities we need to keep track of:

- the pressure in the tube $p(x,t)$ at a given point x along the tube's length and time t, measured in Pa or N/m, and
- the volume velocity in the tube $u(x,t)$ at the same point x along the tube's length and time t, measured in m^3/s.

[1]http://ccrma.stanford.edu/realsimple/waveguideintro/

[2]http://ccrma.stanford.edu/realsimple/travelingwaves/

[3]http://www.kettering.edu/~drussell/Demos/waves/wavemotion.html#longitudinal

It may be shown that the two quantities above obey the wave equation, and thus each quantity may be decomposed into right-traveling and left-traveling wave components as

$$p(x,t) = p^{+}\left(x - \frac{c}{t}\right) + p^{-}\left(x + \frac{c}{t}\right) \tag{1}$$

and

$$u(x,t) = u^{+}\left(x - \frac{c}{t}\right) + u^{-}\left(x + \frac{c}{t}\right), \tag{2}$$

as discussed in the traveling waves laboratory assignment.[4] What the equations above state is that *any* pressure and volume velocity functions that may realistically exist in the tube can each be decomposed into a pair of simpler functions, one a waveform traveling to the right (e.g., $p^{+}(x-c/t)$), and the other a waveform traveling to the left (e.g., $p^{-}(x+c/t)$). The speed at which these waveforms travel, c, is the speed of sound in the tube.

As mentioned earlier, the speed of sound c_{air} in an air-filled acoustic tube is the same as the speed of sound in the open air. As a result, the radius of the tube has no effect on c_{air}. In order to derive a formula for c_{air}, it may first be shown that, based on classical fluid mechanics,

$$c^2 = \frac{\partial p}{\partial \rho}, \tag{3}$$

where p denotes the fluid pressure, and ρ denotes the fluid density.

Next we need to note an important and non-obvious property about sound propagation. It turns out that sound propagates approximately *adiabatically*. What this means is that as the fluid pressure fluctuates rapidly during sound propagation, the temperature of the fluid also fluctuates, but no heat is gained or lost due to this fluctuation. If heat were gained or lost, we would not have an adiabatic process. The key equation describing an adiabatic process relates pressure p and density ρ as follows:

$$p\rho^{-\gamma} = C_0, \tag{4}$$

where C_0 is a constant, and γ is the adiabatic index. For diatomic gases such as air, $\gamma_{\mathrm{dia}} = 1.4$. Using the equations above, it may be shown that

$$c_{\mathrm{air}} = \sqrt{\frac{\gamma_{\mathrm{dia}} p_{\mathrm{air}}}{\rho_{\mathrm{air}}}}. \tag{5}$$

Next, the ideal gas law ($pV = nRT$) may be used to derive the following formula:

$$c_{\mathrm{air}} = \sqrt{\frac{\gamma_{\mathrm{air}} RT}{M_{\mathrm{air}}}}, \tag{6}$$

where R is the universal gas constant (= 8.314 J/[mol K]), T is the absolute temperature in degrees Kelvin, and M is the molar mass of the fluid (in kg/mol). For dry air, $M = 0.02895$ kg/mol.

3.1.1 Problems

1. Using Equation (5) and the ideal gas law, derive Equation (6).
2. At what temperature is the speed of sound in dry air exactly equal to 1 ft/ms?
3. (*Advanced high school problem requiring basic calculus*) Using Equation (3) and Equation (4), derive the formula of Equation (5).

[4] http://ccrma.stanford.edu/realsimple/travelingwaves/

3.2 Wave Impedance, Reflection from a Radius Mismatch

In the previous section, we identified the right- and left-traveling components of two key quantities describing wave propagation in an acoustic tube: the pressure $p(x,t)$ in the tube ($p(x,t) = p^+(x - c/t) + p^-(x + c/t)$), and the volume velocity $u(x,t)$ in the tube ($u(x,t) = u^+(x - c/t) + u^-(x + c/t)$). For the right- and left-traveling components, it turns out we can relate them using relatively simple formulas. Using a combination of calculus, Newton's laws of motion, and the law of conservation of matter, it can be shown that the right-traveling pressure and volume velocity components obey the following formula:

$$p^+\left(x - \frac{c}{t}\right) = R_A\, u^+\left(x - \frac{c}{t}\right), \tag{7}$$

where R_A is the *wave impedance* in the tube, given by the following formula:

$$R_A = \frac{\rho c}{A}, \tag{8}$$

where ρ is the ambient fluid density in the tube, c is the velocity of wave propagation (see Equation (5)), and A is the cross-sectional area of the tube. Thus, the wave impedance R_A relates pressure to volume-velocity everywhere along a plane wave traveling to the right along the axis of an acoustic tube having cross-sectional area A.

Similarly, for the left-traveling wave components, it may be shown that

$$p^-\left(x + \frac{c}{t}\right) = -R_A\, u^-\left(x + \frac{c}{t}\right). \tag{9}$$

It is next interesting to consider what happens to a traveling pressure waveform in an acoustic tube when it encounters a radius mismatch. In other words, what happens when the waveform is traveling through an initial tube with radius r_1, and all-of-a-sudden is transferred into a tube with a second disparate radius r_2? It turns out that part of the waveform will be reflected back into the first tube, and the strength of the reflection k is given by the following formula:

$$k = \frac{R_2 - R_1}{R_1 + R_2}, \tag{10}$$

where R_1 is the wave impedance in the first tube section, and R_2 is the wave impedance in the second tube section. Using the previous formulas, it may be further shown that the reflectance k is also given by the following formula for cylindrical tubes:

$$k = \frac{r_1^2 - r_2^2}{r_1^2 + r_2^2}, \tag{11}$$

where r_1 is the radius of the first tube, and r_2 is the radius of the second tube.

3.2.1 Problems

1. Using the formulas in this section, derive Equation (11).

2. What is the reflectance k when the second tube radius is zero? What does this mean about the reflected wave?

4 Procedure

4.1 Speed of Sound and an Open-Ended Tube

1. Download the `pd` patch `vir_tube1.pd`,[5] and open it in `pd`. Figure 1 shows a sample screen capture of the patch.

2. The patch simulates a single acoustic tube, driven at one end by the stimulus signal shown as `stim` in the patch, and open at the other end. The applied stimulus results in a right-traveling pressure wave along the tube. The tube length, along with the temperature of the air inside the tube, are adjustable using the sliders shown on the patch. Finally, a sensor located at the stimulus-driven end of the tube measures the left-traveling pressure wave response due to `stim`, and stores and displays the result at `resp`.

3. Adjust the temperature to the minimum possible value ($-20°$ C). Next adjust the tube length to approximately 2 ft.

4. If you wish, modify the signal `stim` using the mouse. Ideally, there should be at least one key visible feature of the stimulus signal aligned with the tick marks on the edges of the plot.

5. To launch the pressure-wave stimulus signal into the tube, click the large circular button. You should see the reflected wave in the `resp` signal plot.

6. What do you notice about the polarity of the reflected wave?

7. Using the `stim` and `resp` graphs, estimate the delay between the stimulus and the response signals. Given the length of the tube, calculate an approximate estimate of speed of sound in the tube. What is the percentage error between this estimate and the theoretical value based on the temperature in the tube?

8. Next adjust the temperature in the tube to a different value using the sliders, and repeat the previous three steps. How does an increase in temperature affect the speed of sound in the tube?

4.2 Reflection from a Radius Mismatch

1. Download the `pd` patch `vir_tube2.pd`,[6] and open it in `pd`. Figure 2 shows a sample screen capture of the patch.

2. The patch simulates an acoustic tube driven at one end by the input stimulus signal, `stim`. The length of this tube may be varied using the first horizontal slider on the patch. The other end of the tube is joined to a second cylindrical acoustic tube, whose radius may be adjusted by the second slider on the patch. While the radius of the first tube is set to a constant 10mm, the radius of the second tube may be adjusted to a value between 0mm and 100mm. The second tube is assumed to extend infinitely far to the right. Finally, the temperature may be adjusted using the third slider on the patch.

[5] http://ccrma.stanford.edu/realsimple/vir_tube/vir_tube1.pd
[6] http://ccrma.stanford.edu/realsimple/vir_tube/vir_tube2.pd

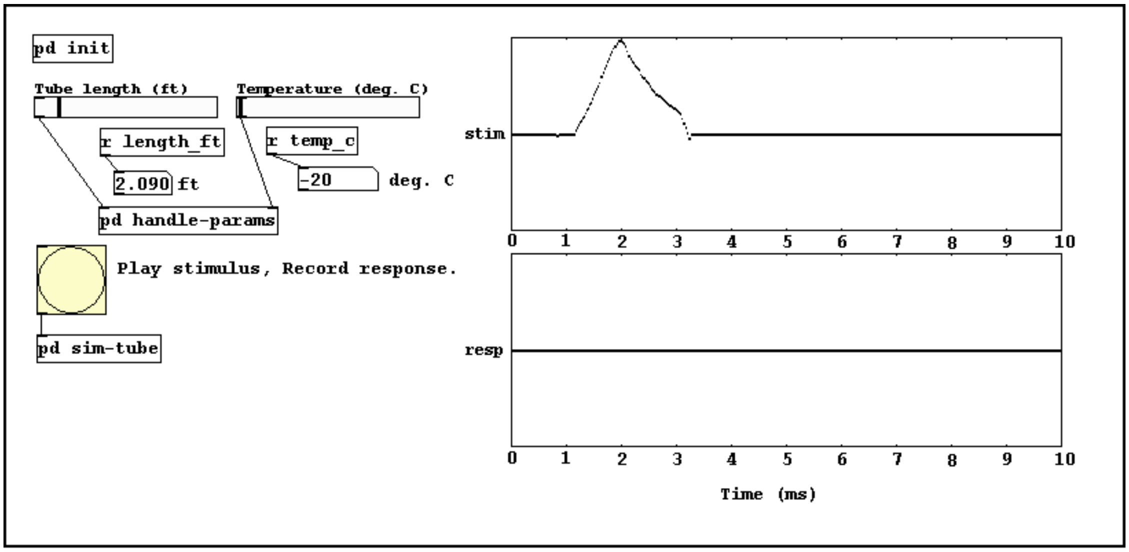

Figure 1: Screen capture showing open-ended acoustic tube patch with variable tube length and air temperature.

3. Adjust the first tube length to its minimum possible value. Next, adjust the temperature to the value at which sound should propagate at approximately 1ft/ms.

4. Using the middle slider, set the second tube radius to 0mm, its minimum possible value. This is one way to simulate closure of the end of the first tube. What is the reflection coefficient for this radius mismatch?

5. Click the large circular button on the patch. You should see a response similar to that of Figure 2. Does the reflected signal agree with your prediction?

6. Next adjust the second tube radius to 100mm, its maximum possible value. Repeat the previous two steps for this new radius mismatch. What situation does this radius mismatch resemble?

7. Adjust the second tube radius to an arbitrary value between the minimum and maximum possible values, and repeat the two steps prior to the previous step.

8. What value for the second tube radius should result in negligible reflection of the stimulus signal? Adjust the second tube radius to this value, and click the large circular button to launch a wave towards the tube junction. Do you achieve negligible reflection as desired?

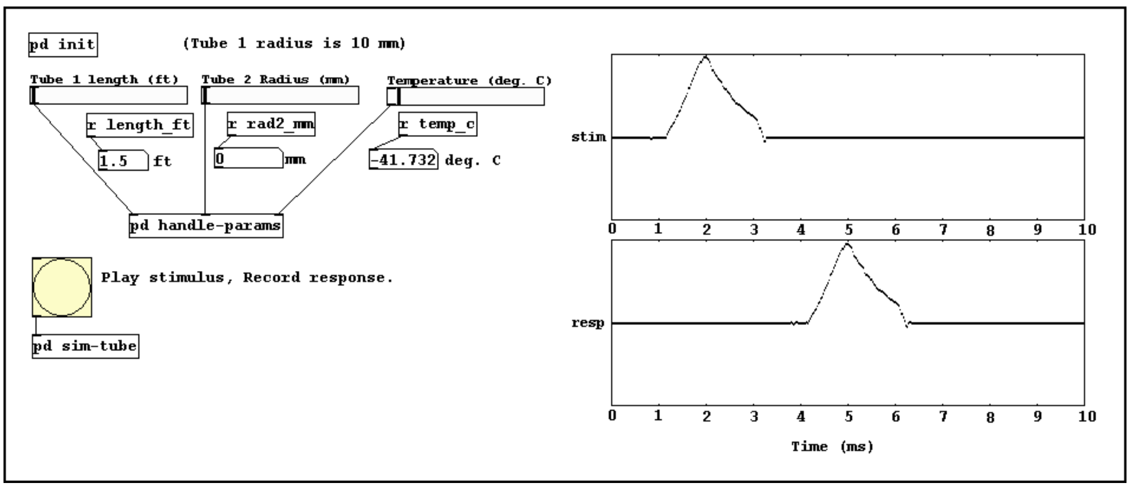

Figure 2: Screen capture showing the cascaded acoustic tube patch with variable first tube length, variable second tube radius, and variable air temperature.