

Harmonic Content of a Plucked String

REALSIMPLE Project*

Edgar J. Berdahl and Julius O. Smith III

Center for Computer Research in Music and Acoustics (CCRMA), and the

Department of Electrical Engineering

Stanford University

Stanford, CA

Abstract

This laboratory assignment should provide the student with an intuitive understanding of the spectral representation of a plucked string sound and how this sound changes as the spectral content is altered. This assignment should be carried out following the first monochord laboratory assignment¹ for the RealSimple Project.²

Contents

1	Introduction	2
2	Summary of Objectives	2
3	Background	2
4	Theory	2
5	Procedure	3
5.1	Part 1: Single Note	3
5.2	Part 2: Melody	5

*Work supported by the Wallenberg Global Learning Network

¹<http://ccrma.stanford.edu/realsimple/lab.inst/>

²<http://ccrma.stanford.edu/realsimple/>

1 Introduction

Any measurement recorded over time is a signal, which may be analyzed using various methods. Such an analysis can aid in understanding the physics underlying what was measured. The analysis is generally made in the time and/or frequency domains. The time domain corresponds to a waveform representation, which is useful for determining the exact times at which events, such as string plucks, occur. A frequency domain analysis is more useful for determining how much energy is present in a sound at various frequencies. For perfectly periodic sounds, the Fourier Series may be used to perform a frequency domain analysis. A Fourier Series demonstration applet may help provide you with some intuition behind using the Fourier Series to construct perfectly periodic sounds. Since the components in a Fourier Series are harmonic, they form a harmonic series.

2 Summary of Objectives

- Intuitively understand what the *spectrum* of a periodic sound is.
- Understand that the *spectrum* of a sound may change over time.
- Learn that the characteristic *timbre* of an instrument may be partially described by its harmonic content, as revealed by the spectrum.
- Investigate the *change in timbre* of a plucked string instrument when the magnitudes of the harmonics are varied.
- Determine how the magnitudes of the harmonics may be altered such that the perceived pitch of the sound increases by an octave.
- *Alter the harmonic content* of the plucked string so that it sounds like another instrument.

3 Background

In the previous laboratory assignment, you measured the velocity of a plucked string and determined that it was not perfectly periodic. Another way of analyzing this signal is to perform a hybrid time and frequency domain analysis such as the Short Time Fourier Transform (STFT). This corresponds to essentially performing a Fourier Series-similar analysis at various windows in time. The end product is a time-varying *spectrum*.

In the previous laboratory assignment, you observed the peaks in the time-varying spectrum. These corresponded to the harmonics or sinusoidal components in the velocity signal, where a large portion of the energy in the string corresponded to very confined (ideally point-like) frequency regions.

4 Theory

Any sound may be decomposed into sinusoids, noise, and transients. For a standard plucked string sound, the sinusoids will usually be approximately harmonically related. This means that the pitch

is roughly the same as the fundamental frequency f_0 . In this case, the frequency $f(n)$ of the n th harmonic is given by the following function:

$$f(n) = nf_0 \tag{1}$$

Imagine hearing a note played on a guitar and the same note played on a clarinet. Even if they were played with the same loudness, you would still be able to distinguish between them because they have different *timbres*. The attributes of timbre are elusive, but the relative magnitudes of the harmonics play an important role. For example, in the lower registers of the clarinet, the magnitudes of the even harmonics tend to be quite small.³ In fact, the second harmonic may be almost completely absent. This is the reason why some people might consider the timbre of the clarinet to be more pure.

In contrast, noise components tend to sound like static, which is the sound made by an old-fashioned radio or television when not tuned to a station. Similarly in speech, noise tends to correspond to *f*-sounds or *s*-sounds. In general, noise components occupy larger, continuous frequency regions in the time-varying spectrum. Transients also occupy larger frequency regions in the spectrum, but they typically correspond to very short events in time such as *t*-sounds in speech or the attack (very beginning) or a plucked string sound.

5 Procedure

5.1 Part 1: Single Note

For this section, a velocity recording of a single pluck on an electric guitar is used. This recording has been split apart into six components. Since the *sum* operation is analogous to the *mix* operation on a mixing board, the components may be summed together yielding the original pluck. The first five components correspond to the first, second, third, fourth, and fifth harmonics (sinusoidal components) of the pluck. The sixth component is the *residual*. It is the difference between the original pluck and the sum of the first five components. This ensures that summing all six components yields the original pluck recording. In an approximate sense, the residual component contains the 6th, 7th, 8th, etc... harmonics of the pluck as well as measurement noise.

1. Download the archive `2-1.zip`⁴, and extract the files included inside it. One of these is the laboratory patch `2-1.pd`, which you should open in `pd`. The other files are audio files, which will be loaded automatically by the patch.
2. Ensure that the patch is not in editing mode, and check the “compute audio” box in the main `pd` window.
3. The volume adjustments are implemented using horizontal sliders to conserve space. This means that when a slider is placed fully to the left, the volume for the corresponding component should be zero. Fully to the right corresponds to a loud volume. Move the “global volume reset” slider so that it is about 25% to the right of the far left. Notice that this resets the volumes of the six channels below. Then click the light-beige “START PLAYING” button. Adjust the global volume reset slider until the volume is comfortable. The six components are being summed to produce the original pluck.

³[http://en.wikipedia.org/wiki/Harmonic_series_\(music\)](http://en.wikipedia.org/wiki/Harmonic_series_(music))

⁴<http://ccrma.stanford.edu/~jos/harmonics/2-1.zip>

4. Check the “LOOP ON” box in the upper left-hand corner of the window. Now click the “START PLAYING” button so that the pluck will be played repeatedly. The spectrum should look similar to one shown in Figure 1.

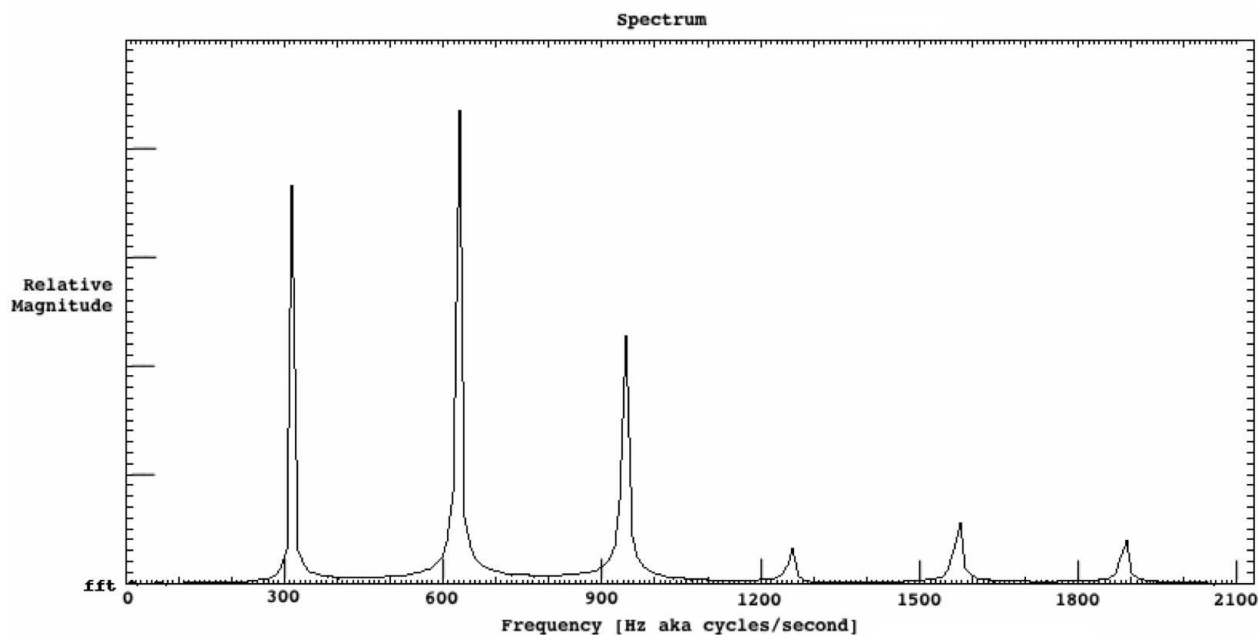


Figure 1: Example spectrum from patch 2-1.pd of part of a single note.

5. What is the fundamental frequency of the plucked string recording?
6. Try reducing the volume of the residual component. How does the sound change? Does it still sound like a real instrument?
7. Use the global volume reset to set the volumes for all of the channels to zero. Try increasing the volume of the first component, listen, and describe the sound. The first harmonic component is approximately an oscillator at the frequency f_0 whose amplitude is varied with time. Repeat the procedure for each of the other components. Describe how the residual component sounds.
8. Experiment with applying different volumes to the various components to see what kind of sounds you can produce.
9. What sets of volumes may you choose to make the note sound as if it had been played an octave higher on the guitar? Mathematically, this means that the new fundamental frequency is $2f_0$. To help you check your answer, we provide sound examples for an Eb_4 note, and then a note an octave higher, which is called Eb_5 .

Eb_4 note on a piano⁵ vs Eb_5 note on a piano (octave higher)⁶

Eb_4 note on an electric guitar⁷ vs Eb_5 note on an electric guitar (octave higher)⁸

⁵<http://ccrma.stanford.edu/jos/realsimple/harmonics/PianoLowEb.wav>

⁶<http://ccrma.stanford.edu/jos/realsimple/harmonics/PianoEbOctave.wav>

⁷<http://ccrma.stanford.edu/jos/realsimple/harmonics/GuitarLowEb.wav>

⁸<http://ccrma.stanford.edu/jos/realsimple/harmonics/GuitarEbOctave.wav>

5.2 Part 2: Melody

Now you will use a different patch. The only difference will be that a whole melody will be played back rather than a single note.

1. Close the laboratory patch 2-1.pd. Do not worry about saving the patch.
2. Download the archive 2-2.zip⁹, and extract the files included inside it. Open the laboratory patch 2-2.pd in pd. Again, the patch will load the other waveforms included in the archive.
3. Ensure that the patch is not in editing mode and that the “compute audio” box in the main pd window is checked.
4. Experiment with applying different volumes to the various components to see what kind of sounds you can produce.
5. Try to find volumes for the components such that the mix sounds more like a clarinet than a guitar. To check your answer, you may listen to this mix¹⁰.
6. What are some reasons why the result does not sound exactly like it had been played on a clarinet?
7. Close the pd patch.

⁹<http://ccrma.stanford.edu/~jos/harmonics/2-2.zip>

¹⁰<http://ccrma.stanford.edu/jos/realsimple/harmonics/ClarinetSynthesized.wav>