

## Resonator Factoring

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### Outline

- Mode Extraction by Inverse Filtering
- Shortened Resonator Impulse Response
- Localized Single-Mode Inverse Filter
- Example for a Guitar Body

## Body Resonator Factoring

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A valuable way of shortening the excitation table in commuted waveguide synthesis is to *factor* the body resonator into its *most-damped* and *least-damped* modes.

- The most-damped modes are then *commuted* and *convolved* with the external excitation.
- The least-damped modes can be left in *parametric form* (recursive digital filter sections)

### Advantages:

- Excitation table is shortened
- Excitation-table signal-to-quantization-noise ratio is improved
- The most important resonances remain *parametric*, facilitating real-time control.
- Multiple body outputs become available (e.g., for more diverse spatialization)
- Resonators are often available in a separate effects unit, making them “free”
- Provides a fairly continuous memory vs. computation trade-off

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\*Work supported by the Wallenberg Global Learning Network

## Mode Extraction Techniques

The goal of resonator factoring is to identify and remove the least-damped resonant modes of the impulse response. This means finding and removing the narrowest “peaks” in the frequency response.

Two Basic Methods:

1. Complex spectral subtraction (equivalent to subtracting a second-order impulse-response)

$$H_r(z) = H(z) - \frac{b_0 + b_1 z^{-1}}{1 + a_1 z^{-1} + a_2 z^{-2}}$$

- Must accurately estimate phase and amplitude, as well as frequency and bandwidth
- Requires resonators in *parallel* with residual  
⇒ residual not readily *commuted* with string

2. Inverse-filtering

$$H_r(z) = H(z) (1 + a_1 z^{-1} + a_2 z^{-2})$$

- Factored resonator components are in cascade (series)
- Residual (damped) modes more easily commuted with the string
- Only need to measure frequency and bandwidth, not amplitude and phase

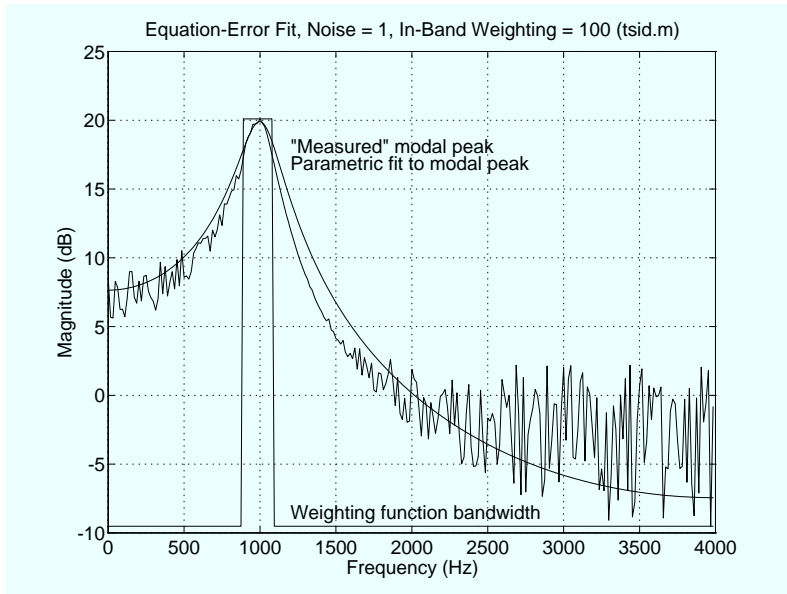
## Mode Extraction by Inverse Filtering

Various methods are applicable for estimating spectral peak parameters:

- (1) Direct amplitude-response peak measurement on FFT magnitude data
  - Center frequency
  - Bandwidth
- (2) Weighted digital filter design
- (3) Linear prediction  
(special case of (2))
- (4) Sinusoidal modeling  
(like (1) but looking across multiple time frames)
- (5) Late impulse-response analysis  
(useable with all methods)
- (6) Work over *Auditory frequency scale* such as the *Bark scale*  
(useable with all methods)

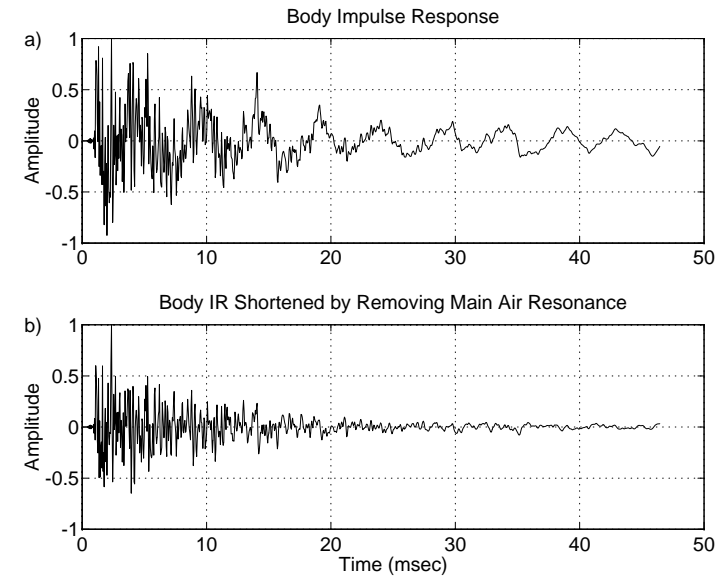
## Example of Body Resonator Factoring

Example of weighted digital filter design using `invfreqz` in Matlab:



## Shortened Body Impulse Response

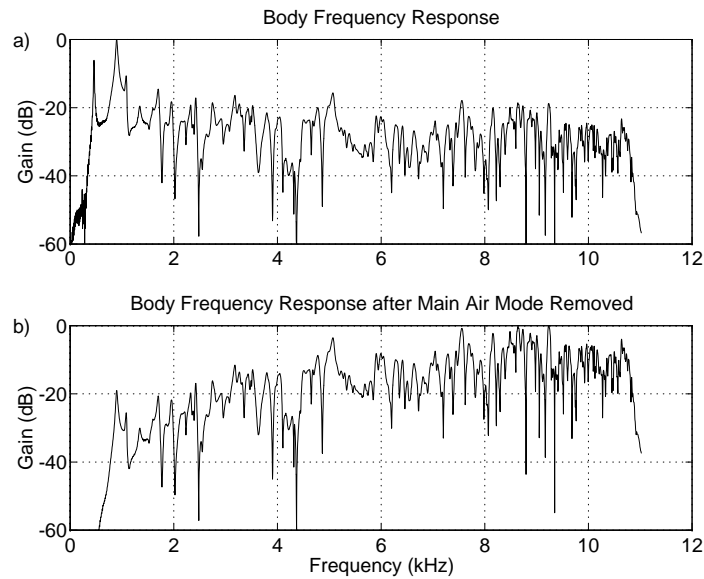
Classical guitar body impulse response body before and after removing the first peak (main Helmholtz air resonance) using a second-order inverse filter:



- Shortened excitation can be truncated to  $\approx 100$  ms
- Shortened excitation can often be replaced by a *filtered noise burst*

## Corresponding Amplitude Response

Normalized Bark-warped amplitude response, classical guitar body, before and after second-order FIR inverse filtering:



## Localized Second-Order Mode Elimination Filter

$$H_r(z) \triangleq \frac{A(z)}{A(z/r)} \triangleq \frac{1 + a_1 z^{-1} + a_2 z^{-2}}{1 + a_1 r z^{-1} + a_2 r^2 z^{-2}}$$

- $A(z)$  = *inverse filter* determined by
  - peak frequency
  - peak bandwidth
- $A(z/r)$  = same polynomial with *roots contracted by  $r$*
- For  $r \approx 1$  (but  $< 1$  for stability), poles and zeros substantially *cancel* far away from the removed mode  $\Rightarrow$  mode removal is *localized*.
- Similar in spirit to *dc blocker*
- $r$  can be interpreted as the *new pole radius* for the “canceled” pole.
- Ideally,  $r$  should equal the radius of the neighboring poles so that all will decay at the same rate (recall late reverb synthesis story)

## Matlab for Localized Peak Removal

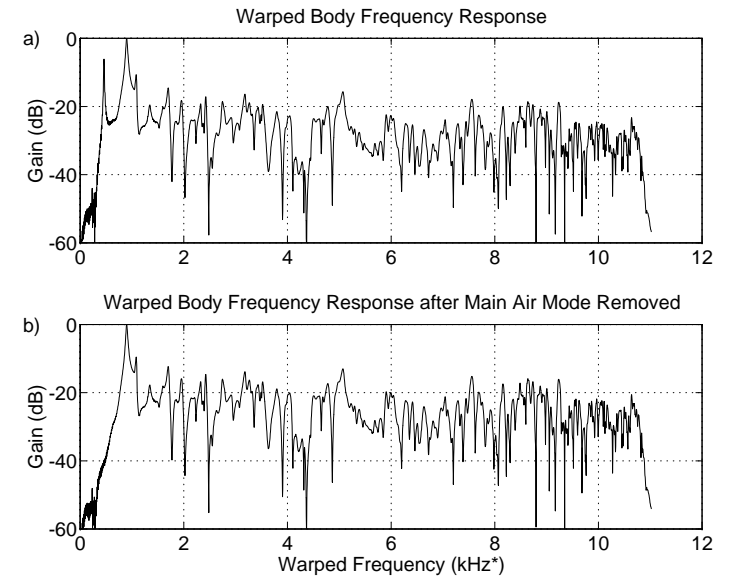
```
freq = 104.98; % estimated peak frequency in Hz
bw = 10;       % peak bandwidth estimate in Hz

R = exp(-pi * bw / fs); % pole radius
z = R * exp(j * 2 * pi * freq / fs); % pole itself
B = [1, -(z + conj(z)), z * conj(z)] % inverse filter numerator
r = 0.9; % zero/pole factor (notch isolation)
A = B .* (r .^ [0 : length(B)-1]); % inverse filter denominator

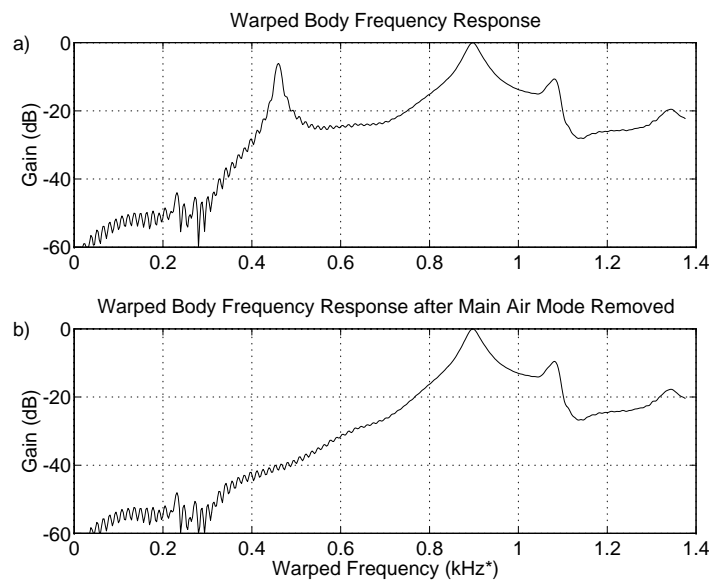
residual = filter(B,A,bodyIR); % apply inverse filter
```

## Localized Peak Removal Example

In this example,  $r = 0.9$  (arbitrary choice - not critical)



## Close-Up on Localized Peak Removal Example



First eighth of previous figure.