Changes added by A. H. Benade to his paper, *On the Mathematical Theory of Woodwind Finger Holes*, J. Acoust. Soc. Am, *32*(12):1591-1608, December 1960.

Benade kept a copy of the above paper in his files to which he added marginal annotations and some corrections. Some are dated, with December 1979 being the latest. Virginia Benade has transcribed the remarks (some of which may not scan well) with help from former Benade student Peter Hoekje, to make a readable addendum to the paper.

page 1593, by Eq. (3):

See C. J. Nederveen and D. W. Wollften Pathe, *Acustica* 13:65 (1963).

page 1593, by Eq. (4):

checks ok

page 1594, left and top margins:

A better calculation for Eq. (7) is gotten by

$$T = \int_0^{l+\Delta l} dx/c = \int_0^{l} dx/v_c$$

$$\Delta l = \int_0^{l} \frac{dx}{(v_c/c)} \cong \int_0^{l} 1/2\ D_c(x)\,dx$$

for flutes with b, t constant

$$\Delta l = \frac{t}{4}\left(\frac{b}{a}\right)^2 \times \frac{\ln\left(\frac{s_j}{s_i}\right)}{\text{"}0.03\text{"}}$$

or, if f_H is frequency played thru highest hole, [and] if f_L is frequency played through the lowest hole,

$$\Delta l = \frac{\frac{1}{4}\left(\frac{b}{a}\right)^2 t \ln\left(\frac{f_H}{f_L}\right)}{0.03}$$

$$\left(\frac{1}{4} \div 0.03 = 83.3\right)$$

page 1594, by Eq. (7):

[Eq. (7) was corrected to read]:

$$\Delta l = l\left\{\frac{r \times [1 - (v_c/c)]}{1 - r[1 - (v_c/c)]}\right\}$$

page 1594, by Eq. (8):

cutoff frequency for woodwinds comes where denominator changes sign.
Good accuracy from low-freq approx:

$$f_c = \frac{c}{\pi}\left(\frac{b}{a}\right)\frac{1}{\sqrt{8t_e s}}$$

If $t \to 0$, and $t_e \to 2b$, then the highest harmonic of a double open pipe that lies below cutoff will be less than N = 5.35(b/a) (this is calculated for limit of $b = s$).

[Added probably later] Eqs. (8) and (9) give same cutoff in long-wavelength approximation.

page 1594, bottom margin:

Dec. '79 [Ink color associates the date with the next 3 lines; and the equation starting m_e seems also to be associated with the Dec '79 entry by way of a double-headed arrow.]

for t_e = 10 mm	Δl_o = 15 mm
b/a = (12/19)	$1 + D_o$ = 1.333
$2s$ = 15 mm	so, $\sqrt{1+D_o} = 1.155$

[More, most likely added later, is found to the right]:

Terminating inertance to match Z_o is $j\dfrac{\rho c}{\pi e^2} = \dfrac{\omega m_e}{c}$

supposing $e = a$, then length m is

$$m_e = \sqrt{1 + D_o}\sqrt{t_e(2s)} \simeq \textit{geom mean of } t_e \textit{ and } 2s$$

[a short section of cylindrical tube is shown, with the cylinder's length identified as m_e and its diameter, as $2e$]

page 1597, Eq. (19c):

[Pencil entries around Eq. (19c) indicate Benade wanted to indicate here the existence of two equations, one for the complete-cone case, the other for incomplete. For clarity, both versions are given here.]

$$\Delta l = -\int_0^l \left(\frac{S_p}{S_0}\right)\left\{+\cos\left[\frac{2n\pi x}{l}\right]+\left(\frac{l}{n\pi x}\right)\sin\left[\frac{2n\pi x}{l}\right]+\left(\frac{l}{n\pi x}\right)^2\sin^2\left[\frac{n\pi x}{l}\right]\right\}dx \qquad \text{(comp. cone)}$$

$$\Delta l = -\int_0^l \left(\frac{S_p}{S_0}\right)\left\{-\cos\left[\frac{2n\pi x}{l}\right]-\left(\frac{l}{n\pi x}\right)\sin\left[\frac{2n\pi x}{l}\right]+\left(\frac{l}{n\pi x}\right)^2\cos^2\left[\frac{n\pi x}{l}\right]\right\}dx \qquad \text{(incomp. cone)}$$

page 1597, Eq. just after (19c):

Note integral → 0 if $(S_p/S_o]$ → const.

page 1599, Eq. (26):

[0.10^3 has been changed to 10^3]

page 1600, near end of paragraph beginning "Once again . . .":

These are in addition to the flattening produced by the mere elongation, which amounts to 12.5¢

page 1600, between the two columns, by the first paragraph of Sec. B:

See C. J. Nederveen article draft for *Acustica* on Clarinet Holes, 10-7-63

[Nederveen sent the draft to Benade. This is in AHB's correspondence files, along with a copy of the published article, Calculations on location and dimensions of holes in a clarinet, *Acustica* 14:227-234 (1964).]

page 1601, Eq. (34)

The final term in Eq. (34) is marked "small," and an asterisk at the end of the equation leads to a comment at the bottom of the page:

for const $\Delta\ell$: $2(b/t_e)(M_e/a)^2\,db = dM_e$

and t_e, for ordinary-size holes: $d(\Delta l) \cong -2t_e(a/b)^2(db/b)$

page 1602

as $\omega \to 0$,

$$\Delta l \to \left\{(v_c/c)\sqrt{1 + D_c}\sqrt{1 + 2(a/b)^2(t_e/s)} - 1\right\}s = s\left\{\sqrt{1 + 2(a/b)^2(t_e/s)} - 1\right\}$$

$$(v_c/c)\sqrt{1 + D_c} = 1$$

[and, in pencil, added later]:

$$\cong \left(2s/\text{arc}\cosh[1 + D_0] - s\right) \leftarrow \text{see Eq. (12)}$$

page 1604, Eqs. (39) and (40):

Equations (39) and (40) both need a minus sign placed in front of the *j* to the right of the equal sign.

page 1605, in margin:

[dated May 28, 1971, which date probably also applies to marginal writing on p. 1606, judging from ink color and density]

Define spectrum conversion coefficient $T(\omega)$, which gives ratio of p_n at mouthpiece of a reed instrument to the *radiated* amplitude from the tone holes.

$$\text{below cutoff } T = \frac{1}{\alpha_n}\sin\left(\frac{\omega}{c}\Delta l_0\right)$$

$$\text{low-frequency approximation } T = \left(\frac{\Delta l_0}{\alpha c}\right)\omega$$

$$\text{above cutoff } T = \text{const}$$

[A simple figure follows, with the upright labeled *T* and the horizontal labeled ω . A straight line rises from the lower left corner at approx 30 degrees, and then sharply changes direction to continue strictly horizontally to the right edge. The sharp change of direction is circled and attached to a comment]:

does it join thus?

[the marginal comment continues]:

Note $\alpha = \omega/c \cot \omega/c\, \Delta\ell_o$ with the final *s* left off in Eq. (38).

page 1606

[with arrow pointing to the heading for Sec. C.]

α is never negligible, because radiation damping plays a role even when the lattice attenuation and boundary losses are negligible